1. **(10 points)** Consider the two lines given by the system of parametric equations $x = t + 2$, $y = 4t - 1$, $z = -5t$ and the system of parametric equations $x = 5s - 1$, $y = 3s$, $z = -s + 3$

   (a) **(3 points)** Are the lines parallel, intersecting, or skew?

   (b) **(7 points)** If the lines intersect, determine the point where it does so; if they do not intersect, determine the distance between the lines.

2. **(15 points)** Answer the following questions about surfaces in space:

   (a) **(5 points)** Determine (either by description or by sketching) the domain of the multi-variable function $f(x, y) = \frac{x^3+\ln y}{\sqrt{x-y}}$.

   (b) **(5 points)** Identify the surface described by the equation $y = 2x^2 - z^2 + 4z$ and state its orientation, if applicable.

   (c) **(5 points)** Give a parametric system of equations describing the curve formed by the intersection of $y = 2x^2 - z^2 + 4z$ and $4x + z = 1$.

3. **(20 points)** Consider the vector-valued function $\mathbf{r}(t) = \left\langle t^2, \frac{2}{3}t^3, t \right\rangle$

(a) **(5 points)** Find the parametric equations of a tangent line to the curve described by $\mathbf{r}(t)$ at the point $(1, -\frac{2}{3}, -1)$.

(b) **(5 points)** Find the arclength along this curve from $(0, 0, 0)$ to $(1, -\frac{2}{3}, -1)$.

(c) **(5 points)** Find the curvature of this curve at $(1, -\frac{2}{3}, -1)$.

(d) **(5 points)** Find the unit binormal vector at $(1, -\frac{2}{3}, -1)$.

4. **(15 points)** In the questions which follow, $\mathbf{u} = \langle 3, -1, 2 \rangle$ and $\mathbf{v} = \langle 0, 3, 4 \rangle$.

   (a) **(4 points)** Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

   (b) **(3 points)** Calculate $\mathbf{u} \times \mathbf{v}$.

   (c) **(4 points)** Identify each of the following four expressions as a vector, a scalar, or as uncalculatable nonsense. You do not need to calculate these expressions or justify your assertions!

      i. $(\mathbf{u} \cdot \mathbf{v})(\mathbf{u} \times \mathbf{v})$.

      ii. $|\mathbf{u}|^2 - (\mathbf{v} \cdot \mathbf{v})$.

      iii. $(\mathbf{u} + \mathbf{v}) \times (\mathbf{u} - \mathbf{v})$.

      iv. $\mathbf{u} + (\mathbf{u} \cdot \mathbf{v})$.

   (d) **(4 points)** Calculate $\text{proj}_{\mathbf{v}}\, \mathbf{u}$.