

1. **(12 points)** Answer the following questions about the function $f(x, y) = \ln(xy - 3y)$.
- (a) **(4 points)** Find all the partial derivatives of f . Label which is which.
- (b) **(4 points)** Find all the *second* partial derivatives of f .
- (c) **(4 points)** Find an equation of the tangent plane to $z = \ln(xy - 3y)$ at the point $(4, 1, 0)$.
2. **(10 points)** Answer the following questions about the function $f(x, y, z) = \frac{2xy-4x^2}{z^2-1}$.
- (a) **(5 points)** Where $u = \frac{2xy-4x^2}{z^2-1}$, calculate $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, and $\frac{\partial u}{\partial z}$.
- (b) **(5 points)** Find an equation of the tangent plane to the level surface $\frac{2xy-4x^2}{z^2-1} = 2$ at the point $(1, 5, -2)$.

3. **(8 points)** Answer the following questions.

(a) **(4 points)** Given $x = \ln(2s - t)$, $y = \arcsin t$, and $u = x^2 + y^2$, calculate $\frac{\partial u}{\partial s}$ and $\frac{\partial u}{\partial t}$; your answers need not be algebraically simplified.

(b) **(4 points)** Calculate $\iint_A (x^2 - 4xy) dA$ if A is the rectangle $1 \leq x \leq 4, 2 \leq y \leq 3$.

4. **(15 points)** Answer the following questions about the rates of change of the function $f(x, y) = xy^2 - 2x^2 + 4y - 7$ at the point $(2, -1)$.

(a) **(5 points)** Find the derivative of $f(x, y) = xy^2 - 2x^2 + 4y - 7$ from the point $(2, -1)$ in the direction $\langle 3, -4 \rangle$.

(b) **(5 points)** Find the direction in which the function $f(x, y) = xy^2 - 2x^2 + 4y - 7$ has the greatest rate of change at the point $(2, -1)$, and identify its rate of change in that direction.

(c) **(5 points)** Identify a direction in which $f(x, y) = xy^2 - 2x^2 + 4y - 7$ has a zero rate of change at the point $(2, -1)$.

5. **(15 points)** Answer the following optimization-related problems.

(a) **(7 points)** Find the critical points of $f(x, y) = 2x^3 - xy^2 + 5x^2 + 4x$, and identify their type.

(b) **(8 points)** Find the three positive numbers x , y , and z whose sum is 100 which maximize the value of x^2yz .