1. **(20 points)** Calculate the following integrals, using whatever approach you find most effective:

   (a) **(3 points)** $\iint_D xy\,dA$ where D is the rectangle with corners $(-2, 1)$, $(-2, 4)$, $(1, 4)$, and $(1, 1)$.

   (b) **(6 points)** $\int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}} x^2 - y^2 dydx$

   (c) **(6 points)** $\iint_D x + ydA$ where D is the region bounded by the curves $x = y^3$ and $x = 2y^2 - y$.

   (d) **(6 points)** $\iiint_E \frac{zy}{x} dV$ where E is the solid bounded by the surfaces $x = 0$, $y = x$, $z = x^2$, and $z = 4$.

2. **(15 points)** Write but do not evaluate the following integrals:

   (a) **(4 points)** A cylindrical integral to calculate the volume of the solid which lies between the sheets of the hyperboloid $x^2 + y^2 - z^2 = -9$ and within the cylinder $x^2 + y^2 = 16$.

   (b) **(4 points)** A spherical integral to calculate $\iiint_E z dV$ where E is the section of the sphere $x^2 + y^2 + z^2 = 25$ where $0 \leq x \leq y$, and $z \geq 0$.

   (c) **(4 points)** A Cartesian (rectangular) form of the integral $\int_{\pi/2}^{\pi} \int_0^{\pi/6} \int_0^{4 \sec \phi} \rho^3 \sin \phi \cos \phi d\rho d\phi d\theta$.

   (d) **(3 points)** A polar integral to calculate $\int_E 2xy dA$, where E is the region given by $4 \leq x^2 + y^2 \leq 16$ with $x \geq 0$ and $y \leq 0$.

3. **(5 points)** Using the transformations $x = \frac{u^2}{v}$, $y = \frac{v}{u}$, evaluate $\iint_D y^2 dA$ over the region D bounded by $xy = 1$, $xy = 2$, $y = 1$, and $xy^2 = 2$.

4. **(6 points)** Determine whether each of the following vector fields as either conservative or nonconservative; for each that is conservative, find a potential function:

- $F(x, y) = e^x \cos y\mathbf{i} - e^x \sin y\mathbf{j}$.

- $G(x, y) = \langle 3x^2 + 2y^2, 4xy + 3\rangle$.

- $H(x, y) = \frac{x}{y}\mathbf{i} + \ln y\mathbf{j}$.

5. **(14 points)** Calculate the following path integrals

   (a) **(4 points)** $\int_C x^2 - y^2 dx$ where C is the curve $y = x^2$ from $(1, 1)$ to $(3, 9)$.

   (b) **(5 points)** $\int_C x^2 ds$ where C is the line segment from $(1, 2)$ to $(0, 5)$.

   (c) **(5 points)** $\int_C F \cdot d\mathbf{r}$, where $F(x, y, z) = y\mathbf{i} + xz\mathbf{j} - 3\mathbf{k}$ and C is the curve given by $y = x^2$ and $z = x^3$ between $(0, 0, 0)$ and $(1, 1, 1)$.