1. **(20 points)** Consider the vector-valued function $\mathbf{r}(t) = \left\langle \frac{t^3}{3}, -2t, t^2 \right\rangle$

   (a) **(5 points)** Find the parametric equations of a tangent line to the curve described by $\mathbf{r}(t)$ at the point $(9, -6, 9)$.

   (b) **(5 points)** Find the arclength along this curve from $(0, 0, 0)$ to $(9, -6, 9)$.

   (c) **(5 points)** Find the curvature of this curve at $(9, -6, 9)$.

   (d) **(5 points)** Find the unit binormal vector at $(9, -6, 9)$.

2. **(10 points)** Consider the line given by the system of parametric equations $x = 3t-5$, $y = -t+2$, $z = 4t + 4$, and the plane given by the equation $2x + 2y - z = 8$.

   (a) **(3 points)** Does the line intersect the plane or not?

   (b) **(7 points)** If the line intersects the plane, determine the point where it does so; if it does not intersect the plane, determine the distance between the line and plane.

3. **(15 points)** In the questions which follow, $\mathbf{u} = 2\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}$ and $\mathbf{v} = \mathbf{i} - \mathbf{k}$.

   (a) **(4 points)** Identify each of the following four expressions as a vector, a scalar, or as uncalculatable nonsense. You do not need to calculate these expressions or justify your assertions!

      i. $(\mathbf{u} \cdot \mathbf{v}) - |\mathbf{v}|$.
      ii. $\frac{\mathbf{u}-\mathbf{v}}{\mathbf{v}}$.
      iii. $|\mathbf{u}|\mathbf{v} - (\mathbf{u} \times \mathbf{v})$.
      iv. $\frac{1}{|\mathbf{u}|}(\mathbf{u} \times \mathbf{v})$.

   (b) **(3 points)** Calculate $\mathbf{u} \times \mathbf{v}$.

   (c) **(4 points)** Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

   (d) **(4 points)** Calculate $\text{proj}_{\mathbf{v}}\, \mathbf{u}$.

4. **(20 points)** Answer the following questions:

   (a) **(5 points)** Evaluate the limit $\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^2+y^2}$ or demonstrate that it does not exist.

   (b) **(5 points)** Given the trajectory $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + 3t\mathbf{k}$, identify the tangential and normal components of the acceleration vector when $t = 2$.

   (c) **(5 points)** Find the equation of the tangent plane to the curve $z = 2x^2 - 3xy$ at $(2, -1, 14)$.

   (d) **(5 points)** Find an equation of the tangent plane to the surface $x^2 + 3y^2 + xz - z^2 = 12$ at the point $(-3, 1, 0)$.

5. **(15 points)** Answer the following questions about optimization:

   (a) **(7 points)** Find the critical points of $g(x, y) = x^2 - 3xy + y^3$ and identify each as a local maximum, local minimum, or saddle point.

   (b) **(8 points)** Find positive x, y, and z such that $xyz = 1$ and which minimize the value of $12x + 9y + 2z$.

6. **(10 points)** Calculate the following integrals:

   (a) **(4 points)** Calculate $\iint_A (xy^2 - \frac{1}{x})dA$ if A is the rectangle $1 \leq x \leq 3, -2 \leq y \leq 1$.

   (b) **(6 points)** Find the integral of xy over the region bounded by the curves $y = x$ and $y = x^4$.

7. **(20 points)** Set up *but do not evaluate* the following integrals.

(a) **(4 points)** An integral to determine the volume of the solid enclosed by the planes $y = 2x$, $y = 0$, $z = 0$, $x = 0$, and $z = 4 - y$.

(b) **(5 points)** A polar iterated form of $\iint_R x^2 - y^2 dA$, where A is the region where $x \geq 0$, $y \geq 0$, and $1 \leq x^2 + y^2 \leq 16$.

(c) **(5 points)** Set up (but do not evaluate) a cylindrical form for the integral $\iiint_E (x+1)dV$ over the solid which lies below the paraboloid $z = 8 - x^2 - y^2$ and above the paraboloid $z = x^2 + y^2$.

(d) **(5 points)** Set up (but do not evaluate) a spherical integral to calculate the volume lying between the cones $z^2 = x^2 + y^2$ and $z^2 = 3x^2 + 3y^2$ for $z \geq 0$, and bounded above by the sphere $x^2 + y^2 + z^2 = 16$.

8. **(10 points)** Calculate the following path integrals:

   (a) **(5 points)** $\int_C x^2 ds$ where C is the line segment from $(0,4)$ to $(3,2)$.

   (b) **(5 points)** $\int_C F \cdot d\mathbf{r}$, where $F(x,y,z) = \langle 4y+z, 3x-z, 2z\rangle$ and C is the curve given by $x=t$, $y=t^2$, and $z=t$ from $(0,0,0)$ to $(2,4,2)$.

9. **(15 points)** Answer the following vector-field-related questions:

   (a) **(4 points)** Determine whether the vector field $G(x,y) = \langle \frac{2x}{y}+3, y^2-\frac{x^2}{y^2}\rangle$ is conservative; if conservative, find a potential function.

   (b) **(7 points)** Use Green's Theorem to evaluate $\int_C x^2y^2dx+8xy^3dy$, where C is the triangular path consisting of linear subpaths from $(0,0)$ to $(1,3)$ to $(0,3)$ and back to $(0,0)$.

   (c) **(4 points)** Find the divergence and curl of the vector field $\mathbf{F}(x,y,z) = 0i + ye^x\mathbf{j} + ye^z\mathbf{k}$