1. **(5 points)** *Find a value of y such that the vectors $\mathbf{u} = \langle 3, 2, 4\rangle$ and $\mathbf{v} = \langle -2, y, 3\rangle$ are perpendicular.*

   In order for two vectors to be perpendicular, their dot product must be 1, so:

$$\begin{aligned} \mathbf{u}\cdot\mathbf{v} &= 0 \\ 3(-2) + 2y + 4\cdot 3 &= 0 \\ 2y &= -6 \\ y &= -3 \end{aligned}$$

2. **(5 points)** *If $\mathbf{a} = \langle 4, -1, -6\rangle$ and $\mathbf{b} = \langle 6, 1, 2\rangle$, calculate $\mathbf{a}\cdot\mathbf{b}$ and $\mathbf{a}\times\mathbf{b}$. Label which is which.*

$$\mathbf{a}\cdot\mathbf{b} = 4\cdot 6 - 1\cdot 1 - 6\cdot 2 = 11$$

$$\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -1 & -6 \\ 6 & 1 & 2 \end{vmatrix} = (-1\cdot 2 + 6\cdot 1)\mathbf{i} + (-6\cdot 6 - 4\cdot 2)\mathbf{j} + (4\cdot 1 + 1\cdot 6)\mathbf{k} = \langle 4, -44, 10\rangle$$

3. **(5 points)** *Find the distance between the plane $3x - 2y + 6z = 6$ and the point $(-1, -1, -1)$.*

   Let us choose an arbitrary point on the plane; one easy selection is $(2, 0, 0)$, although there are many other obvious choices which should give the same result. Then, the vector from $(-1, -1, -1)$ to this point is $\mathbf{u} = \langle 3, 1, 1\rangle$. However, we want the distance *along the normal* from the point to the plane, so we need the length of the projection onto the normal vector $\mathbf{n} = \langle 3, -2, 6\rangle$, which is given by the formula:

$$\text{comp}_{\mathbf{n}}\,\mathbf{u} = \frac{|\mathbf{u}\cdot\mathbf{n}|}{|\mathbf{n}} = \frac{|3\cdot 3 + 1(-2) + 1\cdot 6|}{\sqrt{3^2 + (-2)^2 + 6^2}} = \frac{13}{7}$$

4. **(5 points)** *Calculate the angle between the vectors $\mathbf{u} = \langle 2, -2, 1\rangle$ and $\mathbf{v} = \langle -3, 4, 12\rangle$. Trigonometric and inverse trigonometric functions may be left unreduced.*

   We know that if $\mathbf{u}$ and $\mathbf{v}$ form an angle θ, then $|\mathbf{u}||\mathbf{v}|\cos\theta = \mathbf{u}\cdot\mathbf{v}$; in other words,

$$\theta = \cos^{-1}\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|} = \cos^{-1}\frac{2(-3) - 2\cdot 4 + 1\cdot 12}{\sqrt{2^2 + (-2)^2 + 1^2}\sqrt{(-3)^2 + 4^2 + 12^2}} = \cos^{-1}\frac{-2}{\sqrt{9}\sqrt{169}} = \cos^{-1}\frac{-2}{39}$$

   which, for the curious, is about 93 degrees (which makes sense; a very small negative number points to an angle which is very slightly larger than a right angle).

5. **(2 point bonus)** *Let A be a point not on a line ℓ, and let $\mathbf{u}$ be a vector which is not perpendicular to ℓ. Prove on the back of this page that for any real number k, there is a point B on ℓ such that $\vec{AB}\cdot\mathbf{u} = k$.*

   Consider an arbitrary fixedpoint B_0 on ℓ, and let $mathbf v$ be the any vector parallel to $\mathbf{v}$, so that ℓ can be parametrically represented as the family of points $B_0 + \mathbf{v}t$ for all values of t. Thus, for $B = B_0 + \mathbf{v}t$, we know that $\vec{AB} = \vec{AB_0} + \vec{B_0B} = \vec{AB_0} + \mathbf{v}t$. and thus:

$$\vec{AB}\cdot\mathbf{u} = (\vec{AB_0} + \mathbf{v}t)\cdot\mathbf{u} = \vec{AB_0}\cdot\mathbf{u} + (\mathbf{v}\cdot\mathbf{u})t$$

Then we can solve for the value of t which makes this equal to k, noting that, since $\mathbf{u}$ and ℓ are nonperpendicular, $\mathbf{v} \cdot \mathbf{u} \neq 0$, justifying our last division:

$$\begin{aligned} \vec{AB_0} \cdot \mathbf{u} + (\mathbf{v} \cdot \mathbf{u})t &= k \\ (\mathbf{v} \cdot \mathbf{u})t &= k - \vec{AB_0} \cdot \mathbf{u} \\ t &= \frac{k - \vec{AB_0} \cdot \mathbf{u}}{\mathbf{v} \cdot \mathbf{u}} \end{aligned}$$