1. **(5 points)** *Classify the surface described by the equation* $x^2 + 4y^2 - 3z^2 - 6x + 8y + 12z = 10$*; if applicable, identify its center and orientation.*

   Completing the square gives the equation

$$x^2 - 6x + 9 + 4y^2 + 8y + 4 - 3z^2 + 12z - 12 = 10 + 9 + 4 - 12$$
$$(x-3)^2 + 4(y+1)^2 - 3(z-2)^2 = 11$$

   so this surface is a hyperboloid of one sheet, oriented in the z-direction, with a center as $(3, -1, 2)$.

2. **(6 points)** *Find the arclength of the curve described by* $\mathbf{r}(t) = (t^2 - 2t)\mathbf{i} + \frac{8}{3}t^{3/2}\mathbf{j}$ *between the values* $t = 0$ *and* $t = 4$.

   Note that $\mathbf{r}'(t) = (2t-2)\mathbf{i} + 4\sqrt{t}\mathbf{j}$, so

$$|\mathbf{r}'(t)| = \sqrt{(2t-2)^2 + (4\sqrt{t})^2} = \sqrt{4t^2 - 8t + 4 + 16t} = \sqrt{4t^2 + 8t + 4} = 2t + 2$$

   And then the arclength is given by the integral

$$\int_0^4 |\mathbf{r}'(t)| dt = \int_0^4 2t + 2 dt = t^2 + 2t\Big]_0^4 = 16 + 8 - (0 + 0) = 24$$

3. **(5 points)** *Find an equation (in either parametric or symmetric form) for the* line *tangent to the curve described by* $\mathbf{r}(t) = \langle 6\sqrt{t}, \frac{t^2}{3}, 4t\rangle$ *at the point* $t = 9$.

   Note that $\mathbf{r}'(t) = \left\langle \frac{3}{\sqrt{t}}, \frac{2t}{3}, 4 \right\rangle$. Specifically, $\mathbf{r}(9) = \langle 19, 27, 36\rangle$ and $\mathbf{r}'(9) = \langle 1, 6, 4\rangle$, so the tanglent line at $t = 9$ is a line through the point $(19, 27, 36)$ which is parallel to the vector $\langle 1, 6, 4\rangle$, which yields the parametric equations

$$\begin{cases} x = 19 + t \\ y = 27 + 6t \\ z = 36 + 4t \end{cases}$$

   or the symmetric form

$$x - 19 = \frac{y - 27}{6} = \frac{z - 36}{4}.$$

4. **(4 points)** *Find a vector function which represents the curve of intersection of the surfaces* $z = 4x^2 + y^2$ *and* $y = x^2$.

   Note that this curve must satisfy both of the equations $y = x^2$ and $z = 4x^2 + y^2$; combining the first equation into the second, we get $z = 4x^2 + (x^2)^2 = 4x^2 + x^4$. Since we have y and z in terms of x, we can get a parametric system by letting $x = t$ to get $x = t$, $y = t^2$, and $z = 4t^2 + t^4$. As a vector equation, we thus have

$$\mathbf{r}(t) = \langle t, t^2, 4t^2 + t^4\rangle$$

5. **(2 point bonus)** *Prove on the back of the page that the binormal* $\mathbf{B}(t)$ *is given by* $\frac{\mathbf{r}'(t)\times\mathbf{r}''(t)}{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}$.

We know that $\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$; furthermore, we know that $\mathbf{B}(t)$ is a unit vector (by virtue of being a cross product of perpendicular unit vectors). We know $T(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$. Calculating $N(t)$ is a lot messier, since it involves differentiating $T(t)$:

$$\begin{aligned} T'(t) &= \frac{d}{dt}T(t) = \frac{d}{dt}\left(\frac{1}{|\mathbf{r}'(t)|}\mathbf{r}'(t)\right) \\ &= \frac{1}{|\mathbf{r}'(t)|}\mathbf{r}''(t) + \left(\frac{d}{dt}\frac{1}{|\mathbf{r}'(t)|}\right)\mathbf{r}'(t) \end{aligned}$$

This looks horrific, because $\left(\frac{d}{dt}\frac{1}{|\mathbf{r}'(t)|}\right)$ is pretty messy, but in practice, it ends up being a scalar we don't care about at all, because it vanishes when we take a cross product!

$$\begin{aligned} \mathbf{T}(t) \times \mathbf{N}(t &= \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} \times \frac{1}{|\mathbf{T}'(t)|}\left(\frac{\mathbf{r}''(t)}{|\mathbf{r}'(t)|} + \left(\frac{d}{dt}\frac{1}{|\mathbf{r}'(t)|}\right)\mathbf{r}'(t)\right) \\ &= \frac{1}{|\mathbf{r}'(t)||\mathbf{T}'(t)|}\left(\frac{\mathbf{r}'(t) \times \mathbf{r}''(t)}{|\mathbf{r}'(t)|} + \left(\frac{d}{dt}\frac{1}{|\mathbf{r}'(t)|}\right)(\mathbf{r}'(t) \times \mathbf{r}'(t)\right) \\ &= \frac{\mathbf{r}'(t) \times \mathbf{r}''(t)}{|\mathbf{r}'(t)|^2|\mathbf{T}'(t)|} + \left(\frac{d}{dt}\frac{1}{|\mathbf{r}'(t)|}\right)\mathbf{0} \\ &= \frac{\mathbf{r}'(t) \times \mathbf{r}''(t)}{|\mathbf{r}'(t)|^2|\mathbf{T}'(t)|} \end{aligned}$$

Now, that denominator is awful, but it's a positive number, so we know that $\mathbf{B}(t)$ is a unit vector in the same direction as $\mathbf{r}'(t) \times \mathbf{r}''(t)$; thus, an appropriate thing to divide by is simply $|\mathbf{r}'(t) \times \mathbf{r}''(t)|$ (so, as an added benefit, we now know that $|\mathbf{r}'(t) \times \mathbf{r}''(t)|$ is in fact equal to $|\mathbf{r}'(t)|^2|\mathbf{T}'(t)|$.