1. **(4 points)** Given $f(x,y) = 4x^2 + 2y^3 - 2xy - 1$, find the rate of change of the function at the point $(2,1)$ in the direction $\langle 1, -1 \rangle$.

2. **(5 points)** Find an equation of the tangent plane to the surface $x^2 + 3y^2 + xz - z^2 = 12$ at the point $(-3, 1, 0)$.

3. **(6 points)** Find the critical points of $g(x,y) = x^2 - 3xy + y^3$ and identify each as a local maximum, local minimum, or saddle point.

4. **(5 points)** If $x = e^{t+s}$ and $y = ts - 4t$, and $u = x^2 + y^3$, find $\frac{\partial u}{\partial t}$ and $\frac{\partial u}{\partial s}$; your answers need not be algebraically simplified.

5. **(2 point bonus)** Let $\mathbf{r}(t)$ be a vector-valued function describing a curve in space. Prove that if $\nabla f(x,y,z)$ is perpendicular to $\mathbf{r}'(t)$ at every point on the curve given by $\langle x, y, z \rangle = \mathbf{r}(t)$, then $f(x,y,z)$ is constant on the curve.

Friday, October 15, 2010