1. **(5 points)** *Calculate the integral* $\iint_R \cos(x+2y)dA$ *over the region* $R = \{(x,y)|0 \le x \le \pi, 0 \le y \le \frac{\pi}{2}\}$.

$$\begin{aligned}
\iint_R \cos(x+2y)dA &= \int_0^\pi \int_0^{\pi/2} \cos(x+2y)dydx \\
&= \int_0^\pi \frac{1}{2}\sin(x+2y)\Big]_{y=0}^{y=\pi/2} dx \\
&= \int_0^\pi \frac{\sin(x+\pi)-\sin x}{2} dx \\
&= \frac{-\cos(x+\pi)+\cos x}{2}\Big]_0^\pi \\
&= \frac{-\cos(2\pi)+\cos(\pi)+\cos(\pi)-\cos 0}{2} = \frac{-1-1-1-1}{2} = -2
\end{aligned}$$

The order of the integrations can be reversed, if desired.

2. **(6 points)** *Find the integral of* xy *over the region bounded by the curves* $y = x$ *and* $y = x^4$.

This region lies between $(0,0)$ and $(1,1)$, as can be determined by finding the solutions of $x = x^4$. It is presented as a Type I integral, which is a sound way of approaching it: the "hard" bounds on the left and right are $x = 0$ and $x = 1$, while the nonconstant bounds are $y = x$ above and $y = x^4$ below, so we could set up the integral as such:

$$\begin{aligned}
\iint_D xydA &= \int_0^1 \int_x^{x^4} xydydx \\
&= \int_0^1 x\frac{y^2}{2}\Big]_{y=x^4}^{y=x} dx \\
&= \int_0^1 x\frac{x^2}{2} - x\frac{(x^4)^2}{2} dx \\
&= \int_0^1 \frac{x^3}{2} - \frac{x^9}{2} dx \\
&= \frac{x^4}{8} - \frac{x^{10}}{20}\Big]_0^1 = \left(\frac{1}{8} - \frac{1}{20}\right) - (0-0) = \frac{3}{40}
\end{aligned}$$

Note that the above integral could also be set up as the Type II form $\int_0^1 \int_{\sqrt[4]{y}}^{y} xydxdy$, but is much more difficult to evaluate that way.

3. **(4 points)** *Set up (but do not evaluate) an integral to determine the volume of the solid enclosed by the planes* $y = 2x$, $z = 0$, $x = 0$, *and* $z = 4 - y$.

Looking at the plane which forms the lower bound, $z = 0$, we see that the other three planes intersect that plane in the lines $y = 2x$, $x = 0$, and $y = 4$. Our region in the plane is thus the triangle with coordinates $(0,0)$, $(0,4)$, and $(2,4)$; the only nonconstant bound is the bound on the bottom (if performing a Type I integral) or on the right (if performing a Type II integral) by the line $y = 2x$. It is most natural to set it up as a Type I integral, in which case the left and right bounds are $x = 0$ and $x = 2$, the upper bound is $y = 4$, and the lower bound is

$y = 2x$. Since the height of the solid at a point (x, y) is given by the difference between $4 - y$ and 0, the integrand is $4 - y$, yielding:

$$\int_0^2 \int_{2x}^4 4 - y dy dx$$

Alternatively, we could set up a Type II integral; then the bounds on y are 0 and 4, while the bounds on x are 0 and $\frac{y}{2}$, giving:

$$\int_0^4 \int_0^{y/2} 4 - y dx dy$$

If one *really* wanted to be perverse, even polar coordinates are an option: the line $y = 2x$ corresponds with the boundary $\theta = \arctan 2$, while $x = 0$ corresponds with $\theta = \frac{\pi}{2}$; the bound $y = 4$ is $r \sin\theta = 4$, or $r = 4 \csc\theta$, so we could set up this integral as

$$\int_{\arctan 2}^{\pi/2} \int_0^{4\csc\theta} r(4 - r\sin\theta) dr d\theta$$

4. **(5 points)** *Set up (but do not evaluate) a polar-coordinate iterated form of* $\iint_R x^2 - y^2 dA$, *where A is the region where* $x \geq 0$, $y \geq 0$, *and* $1 \leq x^2 + y^2 \leq 16$.

The condition $1 \leq x^2 + y^2 \leq 16$ describes an annulus (or "ring") with inner radius 1 and outer radius 16. The additional conditions $x \geq 0$, $y \geq 0$ means that we're looking at the quarter-section of this annulus lying in the first quadrant. Fortunately, this shape lends itself easily to polar boundaries; the first quadrant corresponds to $0 \leq \theta \leq \frac{\pi}{2}$, while the annulus in question is given by $1 \leq r \leq 4$, so together with the substitutions $x = r\cos\theta$ and $y = r\sin\theta$, we have an easy construction for our integral:

$$\int_0^{\pi/2} \int_1^4 r[(r\cos\theta)^2 - (r\sin\theta)^2] dr d\theta$$

The order of the integrations can be switched here, if desired; also, the integrand can be simplified to $r^3(\cos^2\theta - \sin^2\theta)$, or even to $r^3\cos(2\theta)$.