1. **(6 points)** *Evaluate the line integral* $\int_C \mathbf{F} \cdot d\mathbf{r}$ *where* $\mathbf{F}(x, y) = e^x\mathbf{i} - 2xy\mathbf{j}$*, and* C *is given by the path* $\mathbf{r}(t) = \langle t, t^2$ *from* $(0, 0)$ *to* $(2, 4)$*.*

   Since the path is given by $\mathbf{r}(t)$ from $t = 0$ to $t = 2$, we have

$$\begin{aligned}\int_C \mathbf{F} \cdot d\mathbf{r} &= \int_0^2 \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t)dt \\ &= \int_0^2 \langle e^t, -2t(t^2)\rangle \cdot \langle 1, 2t\rangle dt \\ &= \int_0^2 e^t - 4t^4 dt = e^t - \frac{4}{5}t^5\Big]_0^2 = e^2 - \frac{128}{5} - (1 - 0)\end{aligned}$$

2. **(3 points)** *Identify the vector field* $\mathbf{F}(x, y) = (1 + 2xy + \sin x)\mathbf{i} + (x^2 - 3)\mathbf{j}$ *as conservative or nonconservative; if conservative, find its potential function.*

   We calculate $\frac{\partial}{\partial y}(1 + 2xy + \sin x) = 2x$ and $\frac{\partial}{\partial x}(x^2 - 3) = 2x$. Since these are equal, the vector field is conservative. Now we calculate the "partial integrals" to find the potential function $f(x, y)$:

$$f(x, y) = \int 1 + 2xy + \sin x dx = x + x^2y - \cos x + C(y)$$

$$f(x, y) = \int x^2 - 3dy = x^2y - 3y + D(x)$$

   Noting that the terms $x - \cos x$ from the first form are subsumed inot the "junk term" $D(x)$ of the second expansion, and likewise $-3y$ is subsumed into the term $C(y)$ of the first expansion, the full expansion is $x + x^2y - \cos x - 3y$ (possibly plus an arbitrary constant).

3. **(7 points)** *Use Green's Theorem to evaluate* $\int_C x^2y^2dx + 8xy^3dy$*, where* C *is the triangular path consisting of linear subpaths from* $(0, 0)$ *to* $(1, 3)$ *to* $(0, 3)$ *and back to* $(0, 0)$*.*

   The described path is a counterclockwise path around the triangular region bounded by $(0, 0)$, $(0, 3)$, and $(1, 3)$, so by Green's Theorem, if the above region is called D:

$$\int_C x^2y^2dx + 8xy^3dy = \iint_D \frac{\partial}{\partial x}(8xy^3) - \partial\partial y(x^2y^2)dA = \iint_D 8y^3 - 2x^2ydA$$

   This area integral can be iterated fairly easily: its left and right bounds are clearly $x = 0$ and $x = 1$, and it is bounded above by the line $y = 3$, but it is bounded below by the line from $(0, 0)$ to $(1, 3)$; that is, $y = 3x$. Thus, our limits can be phrased as such:

$$\begin{aligned}\iint_D 8y^3 - 2x^2ydA &= \int_0^1 \int_{3x}^3 8y^3 - 2x^2ydydx \\ &= \int_0^1 (162 - 9x^2) - (162x^4 - 9x^4)dx \\ &= 162x - 3x^3 - \frac{153}{5}x^5\Big]_0^1 = 159 - \frac{153}{5}\end{aligned}$$

4. **(4 points)** *Find the divergence and curl of the vector field* $\mathbf{F}(x, y, z) = 0i + ye^x\mathbf{j} + ye^z\mathbf{k}$*.*

The curl is calculated as such:

$$\nabla \times F = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & ye^x & ye^z \end{vmatrix} // \quad = \left(\frac{\partial}{\partial y} ye^z - \frac{\partial}{\partial z} ye^x\right)\mathbf{i} + \left(\frac{\partial}{\partial z} 0 - \frac{\partial}{\partial x} ye^z\right)\mathbf{j} + \left(\frac{\partial}{\partial x} ye^x - \frac{\partial}{\partial y} 0\right)\mathbf{k}$$

$$= e^z \mathbf{i} + ye^x \mathbf{k}$$

And the divergence is somewhat easier:

$$\nabla \cdot F = \frac{\partial}{\partial x} 0 + \frac{\partial}{\partial y} ye^x + \frac{\partial}{\partial z} ye^z = e^x + ye^z$$