

Show all work for problems 4–8; use the back of the sheet if necessary.

1. **(1 points)** Express the interval  $(-2, 5]$  in terms of inequalities.

The parenthesis indicates a non-inclusive inequality; the square bracket indicates an inclusive inequality, so this interval can be described as  $-2 < x \leq 5$ .

2. **(1 points)** Express the inequality  $x \geq 6$  in interval notation.

Since this interval includes 6 as its bottom bound, we would use a square bracket on the left boundary. Since there is no bound on the right, we use the peculiar infinite-boundary notation to give interval  $[6, \infty)$  or  $[6, +\infty)$ .

3. **(2 points)** Evaluate the expression  $(-8)^{4/3}$ .

This can be decomposed as  $[(-8)^{1/3}]^4 = \sqrt[3]{-8^4}$ , or as  $[(-8)^4]^{1/3} = \sqrt[3]{(-8)^4}$ . Both will give you the same answer, but the former is much easier to evaluate:

$$\sqrt[3]{-8^4} = (-2)^4 = 16$$

$$\sqrt[3]{(-8)^4} = \sqrt[3]{4096} = 16$$

4. **(3 points)** Expand the polynomial  $2x^3 - (x^2 - 4x)(x + 3)$ .

$$\begin{aligned} 2x^3 - (x^2 - 4x)(x + 3) &= 2x^3 - (x^3 + 3x^2 - 4x^2 - 12x) \\ &= 2x^3 - x^3 - 3x^2 + 4x^2 + 12x \\ &= x^3 + x^2 + 12x \end{aligned}$$

5. **(3 points)** Simplify the rational expression  $\frac{\frac{x+1}{x^2}}{\frac{2x}{x-1}}$ .

$$\frac{\frac{x+1}{x^2}}{\frac{2x}{x-1}} = \frac{x+1}{x^2} \cdot \frac{x-1}{2x} = \frac{x^2-1}{2x^3}$$

6. **(3 points)** Using any method you like, solve the equation  $x^2 + 3x - 4 = 0$ .

You could factor the left side by trial and error, finding that  $(x - 1)(x + 4) = 0$ . Then, using the fact that a product equals zero only if one of its factors equals zero, you may note that  $x - 1 = 0$  or  $x + 4 = 0$ , and thus  $x = 1$  or  $x = -4$ .

Alternatively, you can complete the square (a method not learned in class, but which some of you may have encountered), to build the square  $x^2 + 3x + \frac{9}{4} - \frac{9}{4} - 4 = 0$ , so  $x^2 + 3x + \frac{9}{4} = \frac{25}{4}$ . This gives  $(x + \frac{3}{2})^2 = \frac{25}{4}$ , so  $x + \frac{3}{2} = \pm\sqrt{\frac{25}{4}} = \pm\frac{5}{2}$ . Then  $x = \pm\frac{5}{2} - \frac{3}{2}$ , which takes on values  $x = -4$  and  $x = 1$ .

Another alternative is to use the quadratic formula  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  to get

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot (-4)}}{2 \cdot 1} = \frac{-3 \pm \sqrt{25}}{2} = \frac{-3 \pm 5}{2} = -4 \text{ or } 1$$

7. **(4 points)** *A job pays \$8 per hour for the standard 40 hours worked per week; for time beyond that 40 hours, it pays \$12 per hour. If you earn \$380 at this job in a particular week, how many hours of overtime did you work?*

Let  $x$  represent the number of hours of overtime worked. The amount you earned is the sum of your payment for 40 hours at the standard rate and  $x$  hours at the higher rate: you are thus paid  $40 \cdot 8 + x \cdot 12$  dollars. Since you earned \$380 in total, this question asserts that  $40 \cdot 8 + x \cdot 12 = 380$ . Solving this equation:

$$40 \cdot 8 + x \cdot 12 = 380$$

$$320 + 12x = 380$$

$$12x = 60$$

$$x = 5$$

so this pay results from working five hours of overtime.

8. **(3 points)** *Find an equation, in any of the standard line-equation forms you like, for the line passing through the point  $(-1, 3)$  which is parallel to the line  $y = -2x + 4$ .*

$y = -2x + 4$  has slope  $-2$  by inspection; any line parallel to it will also have slope  $-2$ . We want such a line through the point  $(-1, 3)$ . The easiest way to get an equation for such a line is to invoke the point-slope form

$$(y - y_0) = m(x - x_0)$$

with slope  $m = -2$ , known  $x$ -coordinate  $x_0 = -1$ , and known  $y$ -coordinate  $y_0 = 3$  to get

$$y - 3 = -2(x + 1)$$

If you prefer slope-intercept form, this expression is easily manipulated to give  $y = -2x + 1$ .

It seems like people who could divide fractions easily would have little trouble with their life after that, too.  
—*Only Yesterday*, Isao Takahata