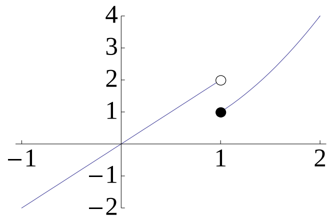
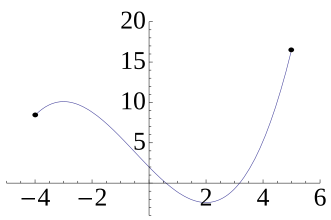
1. **(3 points)** *Sketch a graph of the piecewise function* $f(x) = \begin{cases} 2x \text{ if } x < 1 \\ x^2 \text{ if } x \geq 1 \end{cases}.$

   Note the piecewise jump at the vale $x = 1$:



2. **(3 points)** *Identify the intervals on the following graph on which the function graphed is increasing and the intervals on which it is decreasing. Label which is which.*



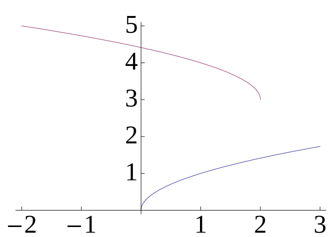
   This graph can be observed to increase on the interval $[-4, -3]$, decrease on $[-3, 2]$, and increase again on $[2, 5]$.

3. **(4 points)** *Determine the average rate of change of the function* $g(z) = z^3 - 3z$ *between* $z = 1$ *and* $z = 3$.

   The rate of change of a function between two values can be determined via the formula:

$$\frac{g(3) - g(1)}{3 - 1} = \frac{(3^3 - 3 \cdot 3) - (1^3 - 3 \cdot 1)}{2} = \frac{(27 - 9) - (1 - 3)}{2} = \frac{20}{2} = 10$$

4. **(5 points)** *The function* $f(x) = \sqrt{x}$ *is shown on the following graph, along with a function* $g(x)$ *which is a transformation of* $\sqrt{x}$. *Find a formula for* $g(x)$.



   We may see that three effects have occurred: the graph has been flipped horizontally, horizontally shifted by 2 units to the right, and vertically shifted by 3 units. Since $f(x) = \sqrt{x}$, we enact the horizontal flip by evaluating $f(-x) = \sqrt{-x}$, then shift right by subtracting 2 from the occurrance of x of the function to get $f(-(x-2)) = \sqrt{-(x-2)} = \sqrt{2-x}$, and then shift upwards by adding 3 to the function to get $g(x) = f(-(x-2)) + 3 = \sqrt{2-x} + 3$.

5. **(5 points)** *Find the vertex,* x*-intercepts if they exist, and* y*-intercept of the quadratic* $g(x) = 2x^2 + 12x + 10$. *Label which is which.*

The square-completion term here would be $\frac{b^2}{4a} = \frac{12^2}{4 \cdot 2} = \frac{144}{8} = 18$, so let us rewrite $g(x)$ in standard form as follows:

$$\begin{aligned} g(x) &= (2x^2 + 12x + 18) - 18 + 10 \\ &= 2(x^2 + 6x + 9) - 8 \\ &= 2(x+3)^2 - 8 \end{aligned}$$

The vertex of this function is thus $(-3, -8)$.

To find the y-intercept, we evaluate $g(0)$ to get $2 \cdot 0^2 + 12 \cdot 0 + 10 = 10$.

Finally, the x-intercepts are the solutions of $g(x) = 0$. Note that since this is a quadratic, it could plausibly have zero, one, or two such solutions. The most direct, if an arithmetically unpleasant route, is the quadratic formula: the solutions to the quadratic equation $2x^2 + 12x + 10 = 0$ are given by

$$\frac{-12 \pm \sqrt{12^2 - 4 \cdot 2 \cdot 10}}{2 \cdot 2} = \frac{-12 \pm \sqrt{144 - 80}}{4} = \frac{-12 \pm \sqrt{64}}{4} = \frac{-12 \pm 8}{4} = -3 \pm 2$$

which gives x-intercepts of -1 and -5.

A mathematician once said that algebra was the science for lazy people—one does not work out x, but operates with it as if one knew it. In our case, x stands for the anonymous masses, the people. Politics mean operating with this x without worrying about its actual nature. Making history is to recognize x for what it stands for in the equation. —*Darkness at Noon*, Arthur Koestler