1. **(6 points)** *Y'Gael sells maps at a price of 40 zorkmids each, and gets 100 customers per day. Market research suggests that every zorkmid by which she raises or lowers the price loses or gains her 5 customers respectively. Determine the daily revenue of her business as a function of price, and determine which price maximizes her revenue.*

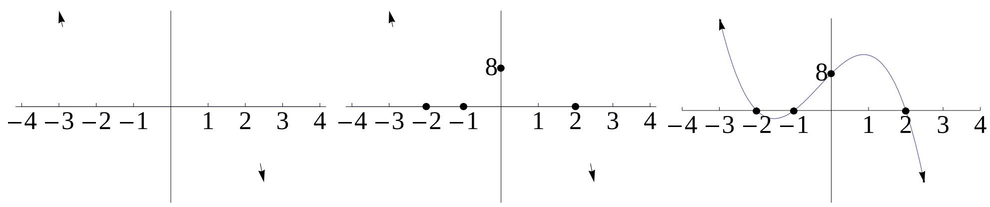
   The demand function at a price x can be seen to be $D(x) = 100-(x-40)\cdot 5 = 300-5x$. Thus, her revenue is the product of the unit price and the number of units sold, so $R(x) = x \cdot D(x) = x(300-5x) = -5x^2 + 300x$.

   The maximum of this quadratic function is its vertex. If we use square-completion with the term $\frac{b^2}{4a} = \frac{300^2}{4(-5)} = -4500$, then:

   $$R(x) = (-5x^2 + 300x - 4500) + 4500 = -5(x^2 - 60x + 900) + 4500 = -5(x-30)^2 + 4500$$

   so the vertex is at $(30, 4500)$, and her best sale price is 30 zorkmids (at which price she will earn Zm4500 total).

2. **(4 points)** *Sketch the function $f(x) = -2(x-2)(x+1)(x+2)$. If you wish, you may label points on the y-axis.*

   Above we see three steps in this process. The first observation is that the highest-degree term in this polynomial is $-2x^3$, which is a negative coefficient and an odd exponent, telling us that it trends from the upper left corner to the lower right, as seen on the arrowheads in the first graph. Then, we take note of this function's zeroes and its easily-calculated y-intercept. We see via the factorization that $f(x) = 0$ when $x = 2, x = -1$, or $x = -2$. In addition, $f(0) = -2(-2)(1)(2) = 8$, so we plot this point as well in the second graph. Finally, we connect up all of our points to get a picture resembling the one in the third graph.

3. **(3 points)** *Find the inverse of the function $h(x) = -5x + 3$.*

   The inverse of $h(x)$ is a function $h^{-1}(x)$ such that $h(h^{-1}(x)) = x$. Thus:

   $$\begin{aligned} h(h^{-1}(x)) &= x \\ -5h^{-1}(x) + 3 &= x \\ -5h^{-1}(x) &= x - 3 \\ h^{-1}(x) &= \frac{x-3}{-5} = \frac{3}{5} - \frac{x}{5} \end{aligned}$$

   The last equality above is purely cosmetic.

4. **(2 points)** *Explain why $f(x) = x^2 - 2x$ has no inverse.*

   There are several examples of diffierent values having the same result under application of this function; one of the most easily discovered is that $f(0) = 0$ and $f(2) = 0$. Thus this function cannot have an inverse, since in order to do so, it would have to be simultaneously the case that $f^{-1}(0) = 0$ and $f^{-1}(0) = 2$.

5. **(5 points)** *Let* $f(x) = \sqrt{x+2}$ *and* $g(x) = \frac{1}{x-1} - 1$*. Write formulas and find the domains for the functions* $(fg)(x)$ *and* $\frac{f}{g}(x)$*. Indicate which is which.*

   It is easy to create formulas for these two functions:

$$(fg)(x) = f(x)g(x) = \left(\sqrt{x+2}\right)\left(\frac{1}{x-1} - 1\right)$$

   Note that the first set of parentheses in the formula is not actually necessary, but it does no harm to include them.

$$\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x+2}}{\frac{1}{x-1} - 1}$$

   This second formula could be cleaned up but there is no good reason to do so.

   In searching for domains, we will need to start by finding the domains of $f(x)$ and $g(x)$ themselves. These are both fairly simple functions, so finding their domains is not too difficult.

   $f(x)$ has in its domain every value where the expression under the radical is non-negative; that is, when $x + 2 \geq 0$, or when $x \geq -2$.

   $g(x)$ has in its domain every value where the denominator of the fraction is nonzero; that is, when $x - 1 \neq 0$, or when $x \neq 1$.

   The domain of $(fg)(x)$ will consist of all values which appear in both of the domains; thus, it consists of all points greater than or equal to -2 *except 1*. In terms of inequalities, this would be written as $x \geq -2$ and $x \neq 1$; in interval notation, it would be the collection of intervals $[-2, 1), (1, \infty)$.

   To find the domain of $\frac{f}{g}(x)$, we need to, in addition to the above procedure, we must determine when $g(x) = 0$. Solving this equality is easy enough: if $\frac{1}{x-1} - 1 = 0$, then $\frac{1}{x-1} = 1$, so $1 = x - 1$ and $x = 2$. The domain of $\frac{f}{g}(x)$ thus resembles the domain of $(fg)(x)$, but we must *additionally* exclude the point $x = 2$, so our domain in inequalities consists of those points satisfying $x \geq -2$, $x \neq 1$, and $x \neq 2$, or, in interval form, the intervals $[-2, 1)$, $(1, 2)$, and $(2, \infty)$.

Mathematicians are like Frenchmen: whatever you say to them they translate into their own language and forthwith it is something entirely different.
—*Maxims and Reflections*, Johann Wolfgang von Goethe