1. **(4 points)** *Write $\frac{2-2i}{1+i}$ in the form $a+bi$.*

   We multiply the numerator and denominator by the conjugate $(1-i)$ of the denominator, in order to get the denominator to be "nice":

$$\frac{(2-2i)(1-i)}{(1+i)(1-i)} = \frac{2-2i-2i-2i^2}{1-i+1-i^2} = \frac{2-4i+2}{1+1} = \frac{-4i}{2} = -2i$$

2. **(3 points)** *Using either synthetic or long division, find the quotient and remainder of the operation $\frac{3x^3-12x^2+10}{x-2}$.*

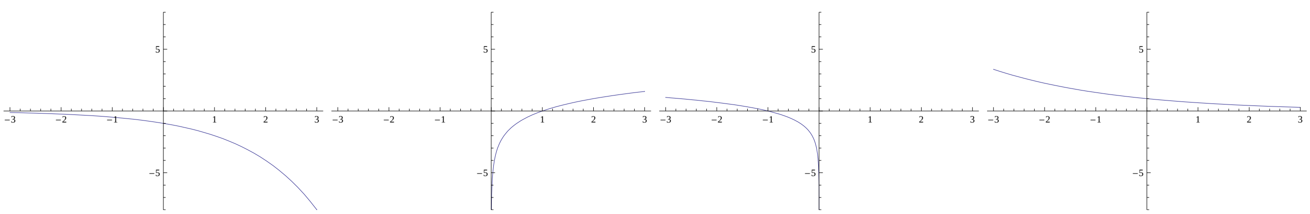
   Here a long division is performed; synthetic division should give the same result:

$$\begin{array}{r|rrrr}
 & 3x^2 & -\ 6x & -12 & \\ \hline
x-2 & 3x^3 & -12x^2 & +\ 0x & +10 \\
 & 3x^3 & -\ 6x^2 & & \\ \hline
 & & -6x^2 & +\ 0x & \\
 & & -6x^2 & +12x & \\ \hline
 & & & -12x & +10 \\
 & & & -12x & +24 \\ \hline
 & & & & -14
\end{array}$$

   And thus the quotient is $3x^2-6x-12$, with a remainder of -14. Thus could also be written as

$$\frac{3x^3-12x^2+10}{x-2} = 3x^2-6x-12-\frac{14}{x-2}$$

3. **(4 points)** *The four following graphs are of the functions $f(x) = -(2^x)$, $g(x) = \left(\frac{2}{3}\right)^x$, $h(x) = \ln(-x)$, and $r(x) = \log_2 x$. Label which one is which.*



   One easy quick identification method would be to note that exponential functions have an easy evaluation at zero, so $f(0) = -(2^0) = -1$ and $g(0) = (\frac{2}{3})^0 = 1$, while logarithms can be easily evaluated at 1, so $h(-1) = \ln(+1) = 0$ and $r(1) = \log_2 1 = 0$. Then it is clear that $f(x)$, $g(x)$, $h(x)$, and $r(x)$ correspond respectively to the graphs containing the points $(0,-1)$, $(0,1)$, $(-1,0)$, and $(1,0)$. These uniquely identify graphs above.

   Alternatively, knowing the shapes of exponential and logarithmic functions is sufficient. $f(x)$ is a negative exponential with exponent larger than 1, so it is a negative growth function; $g(x)$ is a positive exponential with exponent between 0 and 1, so it is a decay function. $h(x)$ and $r(x)$ are logarithmic functions, so they only exist where their argument is positive. For $h(x) = \ln(-x)$, this is true only when x itself is negative, so $h(x)$ is identifiable as a logarithmic-looknig curve which exists only on the left side of the plane; likewise, $r(x) = \log_2 x$ exists when x is positive, so it is a logarithmic-looking curve on the right side of the plane.

4. **(3 points)** *Find the domain of the function $f(x) = \frac{1}{x-2} - \log_{10}(x+3)$.*

   There are two potential problems with evaluatiability of $f(x)$: there is a division, whose denominator must be nonzero, and a logarithm, whose argument must be positive. Thus, it must be the case that $x - 2 \neq 0$ and $x + 3 > 0$. These are simplifiable to $x \neq 2$ and $x > -3$. For those who like interval notation, this means x is defined on the intervals $(-3, 2)$ and $(2, \infty)$.

5. **(3 points)** *Evaluate the following three logarithms:*

   - $\log_5 1$.

     Note that 1 is a power of 5, and, andeed, of any positive number, since $5^0 = 1$. This means that $\log_5 1 = 0$, and, indeed, $\log_b 1 = 0$ for all b.

   - $\log_3 \frac{1}{27}$.

     Trying to express $\frac{1}{27}$ as a power of 3, we recognize the 27, at the least, as 3^3, so $\frac{1}{27} = \frac{1}{3^3}$, and since a reciprocal is like an exponentiation by -1, we can further qualify this as $(3^3)^{-1} = 3^{-3}$. Since $\frac{1}{27}$ can be expressed as this power of 3, we know that $\log_3 \frac{1}{27} = -3$.

   - $\log_{25} 5$.

     Our familiar relationship between the base and the argument of this logarithm would be that $5^2 = 25$; recasting 5 as a power of 25, it wold be more illuminating to note that $5 = \sqrt{25} = 25^{1/2}$. Thus, since whe have expressed 5 as this power of 25, we know that $\log_{25} 5 = \frac{1}{2}$.

6. **(3 points)** *Simplify and evaluate* $2\log_2 \frac{1}{6} - \frac{1}{2}\log_2 25 + \log_2 45$.

   We shall use, in order, the rules for logarithms multiplied by a constant, and the rules for addition and subtraction of logarithms, as such:

$$
\begin{aligned}
2\log_2 \frac{1}{6} - \frac{1}{2}\log_2 25 + \log_2 45 &= \log_2 \left(\frac{1}{6}\right)^2 - \log_2 (25)^{1/2} + \log_2 45 \\
&= \log_2 \frac{1}{36} - \log_2 5 + \log_2 45 \\
&= \log_2 \frac{\frac{1}{36}}{5} + \log_2 45 \\
&= \log_2 \left(\frac{\frac{1}{36}}{5} \cdot 45\right) \\
&= \log_2 \frac{45}{180} = \log_2 \frac{1}{4} = -2
\end{aligned}
$$

> On two occasions I have been asked — "Pray, Mr. Babbage, if you put into the machine wrong figures, will the right answers come out?" In one case a member of the Upper, and in the other a member of the Lower House put this question. I am not able rightly to apprehend the kind of confusion of ideas that could provoke such a question.
>
> —Charles Babbage