1. **(3 points)** *Find the solution of the exponential equation* $2e^{12x} = 17$.

$$\begin{aligned} 2e^{12x} &= 17 \\ e^{12x} &= \frac{17}{2} \\ 12x &= \ln\frac{17}{2} \\ x &= \frac{\ln\frac{17}{2}}{12} \end{aligned}$$

2. **(3 points)** *Find the solution of the exponential equation* $2(3^{2x-1}) + 4 = 58$.

$$\begin{aligned} 2(3^{2x-1}) + 4 &= 58 \\ 2(3^{2x-1}) &= 54 \\ 3^{2x-1} &= 27 \\ 2x - 1 &= \log_3 27 = 3 \\ 2x &= 4 \\ x &= 2 \end{aligned}$$

Note that in the logarithm-taking step we have an exact value because 27 is, in fact, a power of 3.

3. **(4 points)** *The population of Opal City is currently 20000, and the observed relative growth rate is 4.5% per year. Write a function modeling its population* t *years from now, and determine how long it will take for the population to reach 25000.*

Population growth is modeled by $P(t) = P_0 e^{rt}$, where P_0 is the initial population, in this case 20000, and r is the rate of growth, here given to be $4.5\% = 0.045$. Thus, $P(t) = 20000e^{0.045t}$. In light of this model, our above query is simply asking: for what value of t is $P(t) = 25000$?

$$\begin{aligned} 20000e^{0.045t} &= 25000 \\ e^{0.045t} &= \frac{25000}{20000} = \frac{5}{4} \\ 0.045t &= \ln\frac{5}{4} \\ t &= \frac{\ln\frac{5}{4}}{0.045} \end{aligned}$$

As a point of interest, and for context, the above value of t is approximately 5, so specifically it will take nearly 5 years for the population to reach 25000. This contextualization of your result, however, cannot be reasonably accomplished without a calculator.

4. **(4 points)** *The temperature in degrees Fahrenheit of a set of biological samples* t *minutes after being taken out of a deep-freeze is given by the function* $T(t) = 65 - 75e^{-0.1t}$. *The samples will become biologically active when they reach* $20°F$. *How long will it take for this to happen?*

Here we are provided with the model, and simply asked when the modeled quantity is equal to 20°; in other words, we are asked for which value of t it is true that $T(t) = 20$. Plugging this into the known formula for $T(t)$:

$$\begin{aligned} 65 - 75e^{-0.1t} &= 20 \\ -75e^{-0.1t} &= -45 \\ e^{-0.1t} &= \frac{-45}{-75} = \frac{3}{5} \\ -0.1t &= \ln\frac{3}{5} \\ t &= \frac{\ln\frac{3}{5}}{-0.1} \end{aligned}$$

As a point of interest, and for context, the above value of t is approximately 5.1, so specifically it will take slightly more than 5 minutes for the samples to become biologically active. This contextualization of your result, however, cannot be reasonably accomplished without a calculator.

5. **(3 points)** *Find the reference number and terminal point for* $t = \frac{7\pi}{3}$.

   Writing the coefficient of π as a mixed number, we see that $t = \left(2\frac{1}{3}\right)\pi$. Since the nearest multiple of 2 to $2\frac{1}{3}$ is 2 itself, we get a reference number of $\hat{t} = \left(2\frac{1}{3} - 2\right)\pi = \frac{\pi}{3}$. This terminal point, since it is between 0 and $\frac{\pi}{2}$, is in the first quadrant, so its coordinates will be positive. The specific coordinates associated with this reference number we know (from our set of significant reference numbers) to be $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

6. **(3 points)** *Find the reference number and terminal point for* $t = \frac{-41\pi}{4}$.

   Writing the coefficient of π as a mixed number, we see that $t = \left(-10\frac{1}{4}\right)\pi$. Since the nearest multiple of 2 to $-10\frac{1}{4}$ is -10, we get a reference number of $\hat{t} = \left(-10\frac{1}{4} - (-10)\right)\pi = \frac{-\pi}{4}$. This terminal point, since it is between $-\frac{\pi}{2}$ and 0, is in the fourth quadrant, so its x-coordinate will be positive and its y-coordinate negative. The specific coordinates associated with this reference number we know (from our set of significant reference numbers) to be $\left(\frac{\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$.

> Anyone who believes exponential growth can go on forever in a finite world is either a madman or an economist. —Kenneth Boulding