

1. **(2 points)** Find the value of  $\cos \frac{13\pi}{4}$ .

Note that  $\frac{13\pi}{4}$  has the mixed-number representation  $(3\frac{1}{4})\pi$ . The nearest multiple of  $2\pi$  to  $3\frac{1}{4}\pi$  is  $4\pi$ , which we subtract off to get a small angle with the same cosine, namely  $(3\frac{1}{4} - 4)\pi = -\frac{3\pi}{4}$ . This angle can be seen to be in the third quadrant, so the cosine is negative, and the cosine of any odd multiple of  $\frac{\pi}{4}$  is  $\pm\frac{\sqrt{2}}{2}$ ; putting these two facts together, we see that  $\cos \frac{13\pi}{4} = -\frac{\sqrt{2}}{2}$ .

2. **(3 points)** Find the value of  $\csc \frac{-4\pi}{3}$ .

First of all, we note that the cosecant is the reciprocal of the sine, so we might start by determining  $\sin \frac{-4\pi}{3}$ .  $\frac{-4\pi}{3}$  can be rewritten as the mixed-number  $(-1\frac{1}{3})\pi$ . The closest multiple of  $2\pi$  to this is  $-2\pi$ , so we subtract it off to get a small angle with the same cosecant, namely  $(-1\frac{1}{3} + 2)\pi = \frac{2\pi}{3}$ . This angle lies in the second quadrant, so the sine is positive, and thus so is the cosecant. Since this is a multiple of  $\frac{\pi}{3}$  in lowest terms, the sine is  $\pm\frac{\sqrt{3}}{2}$ . Thus,  $\csc \frac{-4\pi}{3} = \csc \frac{2\pi}{3} = \frac{1}{\sin \frac{2\pi}{3}} = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$ .

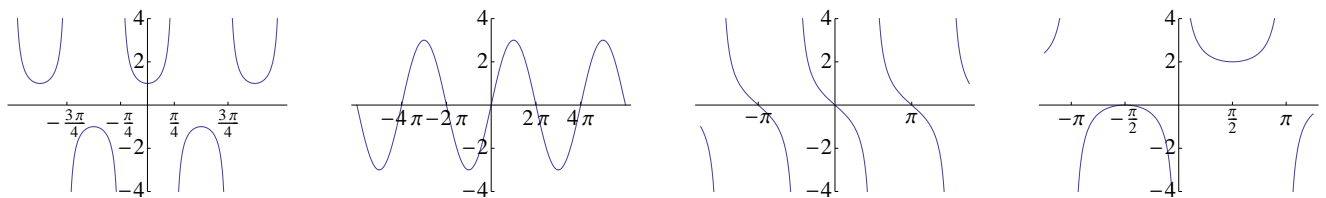
3. **(3 points)** If  $\sin t = -\frac{3}{5}$  and the terminal point determined by  $t$  is in the third quadrant, what is  $\tan t$ ?

We know that  $\cos^2 t + \sin^2 t = 1$ , so  $\cos^2 t = 1 - \sin^2 t = 1 - \frac{9}{25} = \frac{16}{25}$ . Thus,  $\cos t = \pm\sqrt{\frac{16}{25}} = \pm\frac{4}{5}$ . Since  $t$  denotes a point in the third quadrant, both  $\cos t$  and  $\sin t$  are negative, so  $\cos t = -\frac{4}{5}$ , and thus  $\tan t = \frac{\sin t}{\cos t} = \frac{-3/5}{-4/5} = \frac{3}{4}$ .

4. **(2 points)** Given that  $\sin t = \frac{7}{25}$  and  $\cos t = \frac{-24}{25}$ , find the values of the other four trigonometric functions.

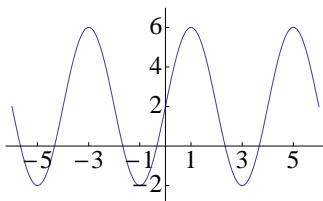
All four other functions can be expressed easily in terms of  $\sin t$  and  $\cos t$ . Specifically,  $\tan t = \frac{\sin t}{\cos t} = \frac{7/25}{-24/25} = -\frac{7}{24}$ , and likewise  $\cot t = \frac{\cos t}{\sin t} = \frac{-24/25}{7/25} = -\frac{24}{7}$ . Also,  $\sec t = \frac{1}{\cos t} = -\frac{25}{24}$  and  $\csc t = \frac{1}{\sin t} = \frac{25}{7}$ .

5. **(4 points)** The following four graphs are of  $f(x) = 1 + \csc x$ ,  $g(x) = -\tan x$ ,  $h(x) = \sec 2x$ , and  $r(x) = 3 \sin \frac{x}{2}$ . Label which is which.



Merely from curve shapes, we can identify 2 of them immediately: the second is sinusoidal and thus matches  $r(x)$ ; the third has the characteristic snakiness of a tangent curve and is thus  $g(x)$ . That leaves only  $f(x)$  and  $h(x)$ , both of which look vaguely secantlike. There are several clues to help differentiate them:  $h(x)$ , with its horizontal compression and lack of a vertical shift, will have a period of  $\pi$  and a “vertical gap” between  $y = -1$  and  $y = 1$ , while  $f(x)$ , with the vertical shift and no horizontal modification, will have a period of  $2\pi$  and a “vertical gap” between  $y = 0$  and  $y = 2$ . This is sufficient information to verify that the first graph is  $h(x)$ , while the last is  $f(x)$ .

6. **(3 points)** Identify the amplitude and period of the curve which is graphed below.



The curve has two consecutive crests at  $x = -3$  and  $x = 1$ , so its period is the distance between them, which is 4. Since the crest is at  $y = 6$  and the trough is at  $y = -2$ , and the amplitude is half this distance, we see that the amplitude is half of 8, or also 4.

7. **(3 points)** *Identify the amplitude, period, and vertical shift of the curve described by the function  $g(x) = -6 \cos(3x) - 2$ .*

The standard cosine curve has amplitude 1 and period  $2\pi$ . This function  $g(x)$  is yielded from the successive operations of a vertical stretch by a factor of 6, a vertical flip, a horizontal compression by a factor of 3, and a vertical shift of  $-2$ . The vertical shift on this function is thus  $-2$ , and the other properties are affected by the stretches and compression: the amplitude is a vertical distance, so stretching the cosine curve's amplitude of 1 by a factor of 6 yields an amplitude of 6. Likewise, the period is a horizontal distance, so compressing the cosine curve's period of  $2\pi$  by a factor of 3 yields a period of  $\frac{2\pi}{3}$ .

Today I am going to give you two examinations, one in trigonometry and one in honesty. I hope you will pass them both, but if you must fail one, let it be trigonometry, for there are many good men in this world today who cannot pass an examination in trigonometry, but there are no good men in the world who cannot pass an examination in honesty.

—Madison Sarratt