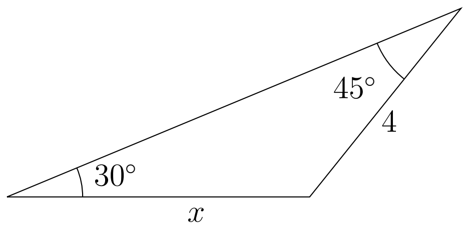
1. **(3 points)** *In the following triangle (not drawn to scale), find the value of x.*



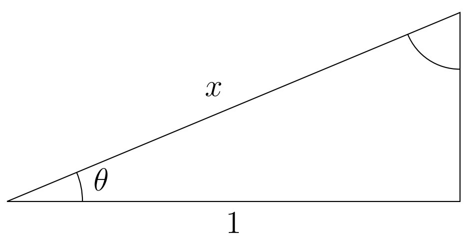
In this triangle two angles have been specified, making it a prime candidate for application of the Rule of Sines. Looknig at the two specified angles and their opposite sides, we get:

$$\frac{x}{\sin 45°} = \frac{4}{\sin 30°}$$

which, multiplying both sides by $\sin 45°$, gives $x = \frac{4 \sin 45°}{\sin 30°} = \frac{4\frac{\sqrt{2}}{2}}{\frac{1}{2}} = 4\sqrt{2}$.

2. **(4 points)** *Simplify* $\cot \operatorname{arcsec}(x)$ *into a form which does not use any trigonometric functions.*

We give a simple name to $\operatorname{arcsec}(x)$; traditionally we might call it θ. To convey this information we might build a triangle exemplifying this relationship between θ and x, which we might write as $\theta = \operatorname{arcsec}(x)$, but more comprehensibly as $x = \sec \theta$; in a right triangle with θ as one of the angles, we know that $\sec \theta = \frac{1}{\cos \theta} = \frac{\text{Hypotenuse}}{\text{Adjacent side}}$. We would thus represent this relationship by making the hypotenuse of the triangle equal x, and the adjacent side be 1, as shown here:



Furthermore, the opposite side, which is not labeled in the above picture, can be calculated by the Pythagorean Theorem to have length $\sqrt{x^2 - 1}$.

After that setup, our actual calculation ends up being very easy! We wanted to find $\cot \operatorname{arcsec}(x)$, which in light of our definition of θ can be written as simply $\cot \theta$. Finding any trigonometric identity of an angle in a labeled right triangle is simply a matter of dividing the appropriate sides: the cotangent is the ratio of the adjacent and opposite sides, so in this case:

$$\cot \theta = \frac{1}{\sqrt{x^2 - 1}}$$

3. **(3 points)** *Evaluate* $\cos 15° \cos 45° - \sin 15° \sin 45°$.

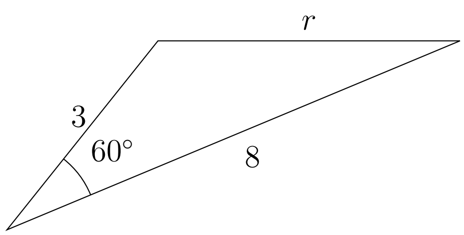
The above expression is a template which is recognizable as one of those covered in our angle-addition rules; recall that

$$\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

whose right side matches nicely with the expression seen here, so

$$\cos 15° \cos 45° - \sin 15° \sin 45° = \cos(15° + 45°) = \cos 60° = \frac{1}{2}$$

4. **(4 points)** *In the following triangle (not drawn to scale), find the value of r.*



Since we have only one labeled angle but know two of the sides, the relationship among the sides is bets discovered by the Rule of Cosines; we use the known angle and the two known adjacent sides to determine that:

$$\begin{aligned} r^2 &= 3^2 + 8^2 - 2 \cdot 3 \cdot 8 \cdot \cos(60°) \\ &= 9 + 64 - 48 \cdot \frac{1}{2} \\ &= 9 + 64 - 24 = 49 \\ r &= \sqrt{49} = 7 \end{aligned}$$

Note that the negative square root can be ignored, since r is a length, and as such is presumably positive.

5. **(4 points)** *Verify the trigonometric identity* $\frac{\cos x}{\sec x \sin x} = \csc x - \sin x$.

We simplify each side of this equality as far as we can, and explore the extent to which the two simplifications can be converted into each other. The left side is

$$\frac{\cos x}{\sec x \sin x} = \frac{\cos x}{\frac{1}{\cos x} \sin x} = \frac{\cos^2 x}{\sin x}$$

while the right, if we think in terms of expressing it as a fraction with $\sin x$ in the denominator, can be manipulated as such:

$$\csc x - \sin x = \frac{1}{\sin x} - \sin x = \frac{1 - \sin^2 x}{\sin x}$$

and now we can invoke the well-known identity $\sin^2 x + \cos^2 x = 1$ to bridge the gap between these two expressions:

$$\frac{1 - \sin^2 x}{\sin x} = \frac{\cos^2 x}{\sin x}$$

6. **(2 points)** *Evaluate* $\arctan(-\sqrt{3})$.

There are a few familiar tangents, and their associated arctangents should be discernable from the familiarity of their form. For instance, we know that $\tan \pm\frac{\pi}{3} = \pm\sqrt{3}$, and thus $\arctan \pm\sqrt{3} = \frac{\pi}{3}$. Thus, this expression evaluates to $-\frac{\pi}{3}$.

The greatest challenge to any thinker is stating the problem in a way that will allow a solution. —Bertrand Russell