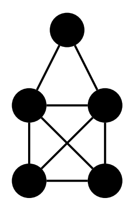
This problem set is due at the beginning of class on *February 4*. Below, "graph" means "simple finite graph".

1. **(10 points)** Let $\tilde{K}_{n,n}$ be the bipartite graph with vertex set $\{a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_n\}$ and containing the edge $\{a_i, b_j\}$ if and only if $i \neq j$.
   (a) **(5 points)** Show that $\tilde{K}_{n,n}$ satisfies the Hall marriage criterion and thus has a perfect matching.
   (b) **(5 points)** How many different perfect matchings are there on $\tilde{K}_{n,n}$ (hint: find the number of perfect matchings on $K_{n,n}$, and exclude those which match some a_i with b_i).

2. **(15 points)** Let $f(G)$ be the number of (not necessarily perfect) matchings on a graph G.
   (a) **(5 points)** Show that for any edge $e = \{u, v\}$ in G, $f(G) = f(G-e) + f(G-u-v)$, where $G - e$ represents G with the edge e removed, and $G - v$ represents G with the vertex v and all incident edges removed, and that $f(G) = 1$ if $\|G\| = 0$.
   (b) **(5 points)** Using the above recurrence, find the number of matchings on the following graph:



   (c) **(5 points)** How could the above recurrence be modified to give the number of *perfect* matchings?

3. **(10 points)** Prove Hall's Theorem by restricting Tutte's Theorem to the bipartite case and exhibiting that the Hall criterion follows from the Tutte criterion if G is bipartite.

4. **(5 points)** Show without using Menger's Theorem that if G is 2-connected and u and v are distinct vertices of G, there is a cycle in G containing both u and v.

A man is about thirty-eight before he stockpiles enough socks to be able to get one matching pair. —*Merrily Harpur*