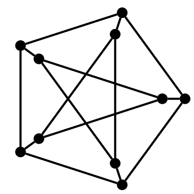
This problem set is due at the beginning of class on *March 23*. Below, "graph" means "simple finite graph" except where otherwise noted.

1. **(10 points)** The *Petersen graph* is shown below.



   (a) **(5 points)** Demonstrate that the Petersen graph is nonplanar by invoking Kuratowski's Theorem.

   (b) **(5 points)** Demonstrate that the Petersen graph is nonplanar by invoking Euler's theorem.

2. **(10 points)** Prove the following results about chromatic number:

   (a) **(5 points)** Show that on any graph G, $\chi(G) \geq \frac{n}{\alpha(G)}$.

   (b) **(5 points)** Show that if G has decomposition into blocks $B_1, B_2, \ldots, B_n$, then $\chi(G) = \max_i(\chi(B_i))$.

3. **(15 points)** A *greedy coloring* of a graph with an ordered set of vertices $v_1, v_2, \ldots, v_n$ is produced by labeling each vertex in order with the lowest number not already used at an adjacent labeled vertex.

   (a) **(5 points)** Show that, for any graph G, there is some ordering of the vertices on which the greedy coloring uses only $\chi(G)$ colors.

   (b) **(5 points)** Demonstrate that for any $k > 2$ there is a bipartite graph G on $2k$ vertices and vertex ordering thereon in which the greedy coloring uses k colors, even though $\chi(G) = 2$.

   (c) **(5 points)** Show that a greedy coloring of the cycle C_n, regardless of the value of n, does not use more than 3 colors.

4. **(5 points)** Prove that a bipartite graph on $n > 2$ vertices is planar only if it has $2n - 4$ or fewer edges.

5. **(5 point bonus)** Prove that for every value of $k > 2$, there is a tree and ordering of the vertices thereon such that a greedy coloring (see above) of the tree requires k colors. What is the smallest such tree you can find?