No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified. Algebraic and trigonometric simplification of answers is generally unnecessary.

1. **(14 points)** Answer the following questions about approximation:

   (a) **(7 points)** Choose $x_0 = 2$ to be an initial approximation of $\sqrt[4]{20}$. Use one step of Newton's method on an appropriately chosen polynomial function to develop x_1, a better rational approximation of $\sqrt[4]{20}$; also give an arithmetic expression (which need not be simplified) for the better approximation x_2 arising from a second step of Newton's method.

   (b) **(7 points)** Starting with an initial value of 1, use two iterations of Newton's method to approximate a zero of $f(x) = x^5 - x^2 - 2x + 5$. Your answer need not be arithmetically simplified.

2. **(20 points)** You wish to fence a 4800-square-foot rectangular field and subdivide it into six plots by placing two internal fences parallel to one pair of sides, and one internal fence parallel to the other pair of sides. What dimensions for the field will use the least quantity of fencing?

1	/ 14
2	/ 20
3	/ 24
4	/ 22
5	/ 20
6	/ (5)
Σ	/100

3. **(24 points)** Answer the following questions related to the shape of the graph of $g(x) = e^x(x-4)$.

   (a) **(4 points)** When is $g(x)$ equal to zero? What is its y-intercept? Label which is which.

   (b) **(7 points)** Where is it increasing? Where is it decreasing? Label which is which.

   (c) **(6 points)** What are its critical points, and is each a local maximum, a local minimum, or neither?

   (d) **(7 points)** Where is it concave up? Where is it concave down? Label which is which. Where, if anywhere, are its points of inflection?

4. **(22 points)** Evaluate the following limits; if they cannot be evaluated, show why not.

(a) $\lim_{x\to 0} \frac{x^2}{1-\cos x}$.

(b) $\lim_{\theta\to 0} \frac{\tan\theta}{\cos\theta}$.

(c) $\lim_{t\to 0} \frac{e^{6t}-e^{2t}}{t^2}$.

(d) $\lim_{x\to +\infty} \frac{\ln(x+3)}{x^2}$.

(e) $\lim_{u\to 1} \frac{u^4-1}{\ln u}$.

5. **(20 points)** Answer the following questions:

   (a) **(7 points)** Find $g(t)$ given that $g'(t) = t - \frac{1}{t^3}$ and $g(1) = 6$.

   (b) **(6 points)** Determine a region whose area is $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{7}{n}\right) \sqrt{1 + \frac{7i}{n}}$.

   (c) **(7 points)** Find the general antiderivative of $h(\theta) = 2\cos\theta + \sec^2\theta - \frac{4}{\theta} + \frac{6}{\sqrt[3]{\theta}}$.

6. **(5 point bonus)** We have seen two procedures for estimating a difficult-to-calculate $\sqrt[n]{k}$: we can either choose a close to $\sqrt[n]{k}$ and perform a linear approximation, or choose a reasonable guess x_0 and perform Newton's method on an easy-to-manage polynomial which has $\sqrt[n]{k}$ as a zero. Prove on the back of this sheet that, in general, if $x_0 = a$, a single step of Newton's method gives the exact same result as the linear approximation.