

Any answers which require logarithms to be expressed should be put in terms of natural or common logarithms. Show all work.

1. **(8 points)** There are currently 5000 inhabitants of the luckless town of Dunwich, MA, and its population decreases by 8% each year due to emigration, death, and mysterious disappearances.

- (a) **(4 points)** Construct a function $f(t)$ to describe the population of Dunwich in t years.

Since the population each year becomes 92% of its former value, after one year the population would be $5000 \cdot 0.92$, after two years it would be $5000 \cdot 0.92 \cdot 0.92$, and so forth. Thus after t years it has gone through this depletion procedure t times, resulting in a population of $f(t) = 5000(0.92)^t$.

- (b) **(4 points)** The remaining denizens of the town will abandon it when there are only 200 people left. When will this occur?

We are solving for when $f(t) = 200$, as such:

$$\begin{aligned} 5000(0.92)^t &= 200 \\ (0.92)^t &= \frac{200}{5000} = \frac{1}{25} \\ t &= \log_{0.92} \frac{1}{25} = \frac{\ln \frac{1}{25}}{\ln 0.92} \end{aligned}$$

This would evaluate to approximately 38.6 years, if calculated; such a calculation is neither necessary nor feasible under assessment conditions.

2. **(3 points)** Find the equation of the line through the points $(-3, 6)$ and $(-1, 10)$.

The slope of this line is given by $m = \frac{10-6}{-1-(-3)} = \frac{4}{2} = 2$. Thus the equation of the line is $y = 2x + b$ for some b . Plugging in either of the known points on the line lets us solve for b :

$$\begin{aligned} 6 &= 2(-3) + b \\ 6 &= -6 + b \\ 12 &= b \end{aligned}$$

so the resulting equation is $y = 2x + 12$.

3. **(6 points)** Identify the domains of the following functions:

- (a) **(3 points)** $f(x) = \frac{2x^3-5}{x^2+x-6}$.

This function contains a fraction, so the denominator must be nonzero; we thus have the condition $x^2 + x - 6 \neq 0$. Using the quadratic formula, it thus follows that x is not equal to either value of $\frac{-1 \pm \sqrt{1+24}}{2} = -3, 2$. Thus $x \neq -3$ and $x \neq 2$, or, in interval form, x is in $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$.

- (b) **(3 points)** $g(t) = \sqrt{3-t} - \sqrt{2+t}$.

We have two square roots appearing in this function, so both of them must have non-negative arguments in order for the function to be evaluatable. Thus, it is necessary that $3-t \geq 0$ and $2+t \geq 0$; from these we respectively derive the conditions $t \leq 3$ and $t \geq -2$, so $-2 \leq t \leq 3$, or, in interval form, t must lie in $[-2, 3]$.

4. (3 points) If $f(x) = 3x^2 - 2x$, simplify the expression $f(a + h) - f(a)$.

$$\begin{aligned} f(a + h) - f(a) &= [3(a + h)^2 - 2(a + h)] - [3a^2 - 2a] \\ &= (3a^2 + 6ah + 3h^2 - 2a - 2h) - (3a^2 - 2a) \\ &= 6ah + 3h^2 - 2h \end{aligned}$$