1. **(4 points)** *Approximate the two following values with well-chosen linear approximations:*

   - $\sqrt[3]{997}$.
     We consider the function $f(x) = \sqrt[3]{x} = x^{1/3}$, whose derivative is $f'(x) = \frac{1}{3}x^{-2/3}$. For x close to 1000 (as 997 is), we can use the linear approximation:

     $$f(x) \approx f(1000) + (x - 1000)f'(1000)$$

     Since $f(1000) = 10$ and $f'(1000) = \frac{1}{3\sqrt[3]{1000}^2} = \frac{1}{300}$, it follows that

     $$f(997) \approx 10 - 3 \cdot \frac{1}{300} = 10 - 0.01 = 9.99$$

     For purposes of comparison, the actual value of $\sqrt[3]{997}$ is around 9.98998998.

   - 3.04^4.
     We consider the function $f(x) = x^4$, whose derivative is $f'(x) = 4x^3$. For x close to 3 (as 3.04 is), we can use the linear approximation:

     $$f(x) \approx f(3) + (x - 3)f'(3)$$

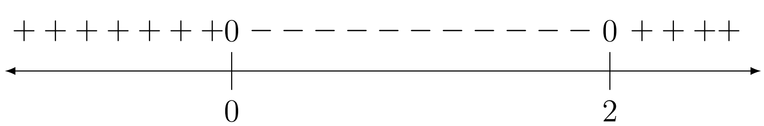
     Since $f(3) = 81$ and $f'(3) = 4 \cdot 27 = 108$, it follows that

     $$f(3.04) \approx 81 + 0.04 \cdot 108 = 81 + 4.32 = 85.32$$

     For purposes of comparison, the actual value of 3.04^4 is around 85.40717.

2. **(10 points)** *Let $f(x) = 2 + 3x^2 - x^3$. Answer the following questions about this function.*

   (a) **(4 points)** *Where is it increasing? Where is it decreasing?*
   We calculate $f'(x) = 6x - 3x^2$; this is a polynomial so it is defined everywhere, and since $f'(x) = 3x(2 - x)$, it is evident that $f'(x) = 0$ at $x = 0$ and $x = 2$. Now we can probe in the three regions: when $x < 0$, when $0 < x < 2$, and when $x > 2$, with the three following respective calculations: $f'(-1) = 6(-1) - 3(-1)^2 = -9$, $f'(1) = 6(1) - 3(-1)^2 = 3$, and $f'(3) = 6(3) - 3(3)^2 = -9$, so the diagram of the sign of $f'(x)$ at various values is given by:

   (b) **(3 points)** *Where are its local maxima and minima?*

   (c) **(3 points)** *Where is it concave up, where is it concave down, and what are its points of inflection?*

3. **(6 points)** Calculate the following limits:

   (a) $\lim_{x\to 0} \frac{x - \sin x}{x - \tan x}$.

   (b) $\lim_{t\to 3} \frac{t^2 - 2t - 3}{t^2 + 2t + 2}$.

4. **(2 point bonus)** Find an example of a continuous function with infinitely many local maxima, infinitely many global minima on $(-\infty, \infty)$, and only one global maximum.