

This test is closed-book and closed-notes. No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified.

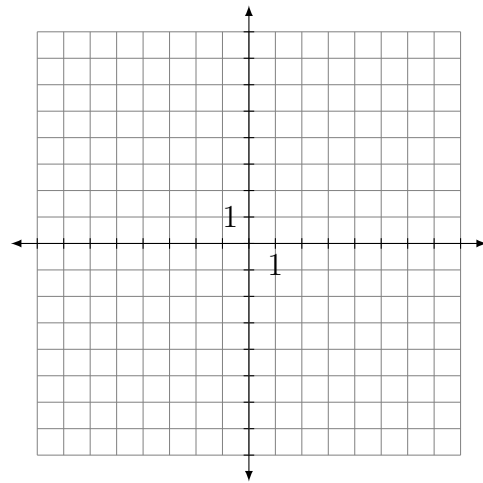
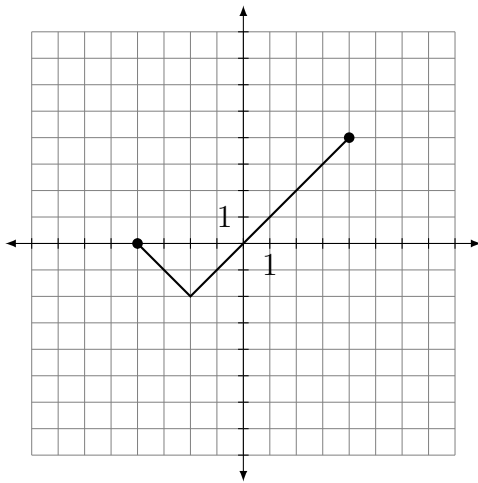
The problems are in no particular order, and it is suggested that you look at all of them before beginning to answer any.

1. (10 points) Answer the following questions about graphs.

(a) (3 points) Given the piecewise function $g(x) = \begin{cases} x^2 - 2 & \text{if } x > 1 \\ 2x - 5 & \text{if } x \leq 1 \end{cases}$, calculate the following values:

- $g(3)$.
- $g(2)$.
- $g(1)$.

(b) (4 points) For $f(x)$ as shown on the graph on the left, sketch the graph of its transformation $g(x) = 2f(-x) - 1$ on the right.



(c) (3 points) Determine the equation of the line through the points $(1, 4)$ and $(4, -5)$.

1	/10
2	/10
3	/15
4	/15
5	/10
Σ	/60

2. **(10 points)** The Hong Kong Cavaliers have 1000 fans who would come to a concert if tickets cost \$10. Polling of the fanbase suggests that every increase in the ticket price by \$1 would reduce attendance by 50.

(a) **(4 points)** Express the number of concertgoers as a function of the ticket price.

(b) **(2 points)** Express the total revenue from ticket sales as a function of the ticket price.

(c) **(4 points)** Find the ticket price which maximizes revenue.

3. **(15 points)** Answer the following questions about the functions $f(x) = \frac{2}{x+1}$ and $g(x) = \frac{x}{x+2}$. In each question asking for multiple answers, *label which is which*.

(a) **(3 points)** Find the inverse of the function $f(x)$.

(b) **(2 points)** Write formulas, which need not be simplified, for $(g - f)(x)$ and $\frac{f}{g}(x)$.

(c) **(3 points)** Write formulas, which need not be simplified, for $f(g(x))$ and $f(f(x))$.

(d) **(3 points)** Determine the domains of $f(x)$ and $g(x)$.

(e) **(4 points)** Determine the domains of $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$, and $\frac{f}{g}(x)$.

4. **(15 points)** Perform the following arithmetic and algebraic operations.

(a) **(3 points)** Factor the quadratic $x^2 - 8x + 12$.

(b) **(3 points)** Expand and simplify the polynomial $(x^3 + 5) - (2x - 1)(x^2 + 3x)$.

(c) **(3 points)** Calculate $16^{3/4}$.

(d) **(3 points)** Simplify the expression $\frac{x}{x-4} - \frac{3}{x+6}$.

(e) **(3 points)** Simplify the expression $\frac{(2x^3)^2(3x^4)}{(x^3)^4}$.

5. **(10 points)** Answer the following questions about the quadratic $q(x) = 6x^2 + 12x - 5$.
- (a) **(2 points)** What is the average rate of change of the function $q(x)$ between the points $x = -1$ and $x = 1$?
- (b) **(3 points)** Put the quadratic $q(x)$ in standard form.
- (c) **(1 point)** Does $q(x)$ have a maximum or minimum value? If so, identify which it is and what its value is.
- (d) **(4 points)** Determine the vertex of this quadratic function, its x -intercepts if they exist, and its y -intercept. Label which is which.