This test is closed-book and closed-notes. No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified.

The problems are in no particular order, and it is suggested that you look at all of them before beginning to answer any.

1. **(10 points)** You are placing a fence around all four sides of a farmyard, as well as a fence parallel to the front fence running down the middle. You have 600 feet of fencing.

    (a) **(4 points)** Find the length of the side fences as a function of the length of the front fence.

    (b) **(2 points)** Find the area of the yard as a function of the length of the front fence.

    (c) **(4 points)** Find the dimensions of the yard which maximize its area.

2. **(15 points)** Perform the following arithmetic and algebraic operations.

(a) **(3 points)** Simplify the expression $\frac{x}{x-1} + \frac{2}{x-2}$.

(b) **(3 points)** Calculate $27^{-4/3}$.

(c) **(3 points)** Factor the quadratic $x^2 - 7x - 8$.

(d) **(3 points)** Expand and simplify the polynomial $(x^2 + 1)(2x - 4) - (x^3 + 2)$.

(e) **(3 points)** Simplify the expression $\left(\frac{x^3 y^2}{z}\right)^4 \left(\frac{xz^2}{y^3}\right)$.

3. **(15 points)** Answer the following questions about the functions $f(x) = \frac{2}{x+4}$ and $g(x) = x^2 - 9$. In each question asking for multiple answers, *label which is which.*

(a) **(3 points)** Determine the domains of $f(x)$ and $g(x)$.

(b) **(2 points)** Write formulas, which need not be simplified, for $(f - g)(x)$ and $\frac{f}{g}(x)$.

(c) **(4 points)** Determine the domains of $(f - g)(x)$ and $\frac{f}{g}(x)$.

(d) **(3 points)** Write formulas, which need not be simplified, for $f(g(x))$ and $g(g(x))$.

(e) **(3 points)** Find the inverse of the function $f(x)$.

4. **(10 points)** Answer the following questions about the quadratic $s(x) = -3x^2 - 12x - 12$.

   (a) **(3 points)** Put the quadratic $s(x)$ in standard form.

   (b) **(1 point)** Does this function have a maximum or minimum value? If so, identify which it is and what its value is.

   (c) **(4 points)** Determine its vertex, x-intercepts if they exist, and y-intercept.

   (d) **(2 points)** What is the average rate of change of the function $s(x)$ between the points $x = 0$ and $x = 2$?

5. **(10 points)** Answer the following questions about graphs.

   (a) **(3 points)** Determine the equation of the line through the point $(-1, 2)$ which is perpendicular to $y = 2x + 4$.

   (b) **(3 points)** Draw the graph of the piecewise function $g(x) = \begin{cases} x - 1 \text{ if } & x \leq -1 \\ x + 1 \text{ if } & x > -1 \end{cases}$
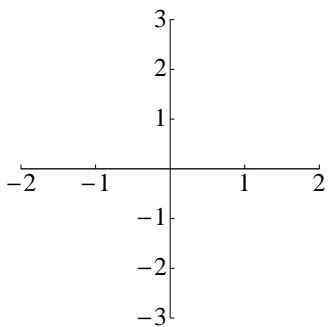
   (c) **(4 points)** The graph $f(x) = |x|$ is shown, together with a transformation $g(x)$. Find a formula for $g(x)$.