

This test is closed-book and closed-notes. No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified.

The problems are in no particular order, and it is suggested that you look at all of them before beginning to answer any.

1. **(10 points)** Market research suggests that if tickets to a concert are \$5, there will be 500 tickets sold, but every dollar the price is increased will lose 20 customers.

(a) **(3 points)** Find a function describing the *demand* for tickets as a function of price.

(b) **(3 points)** Find a function describing the total *revenue* from ticket sales as a function of price.

(c) **(4 points)** Find a sale price for tickets which maximizes revenue, and the total revenue earned at this price. Label which is which.

1	/10
2	/15
3	/12
4	/13
5	/ 6
6	/14
7	/10
8	/10
Σ	/90

2. **(15 points)** Answer the following questions about the functions $f(x) = \frac{\sqrt{x}}{x-6}$ and $g(x) = \log_2(x-3)$. In each question asking for multiple answers, *label which is which*.

(a) **(3 points)** Determine the domains of $f(x)$ and $g(x)$.

(b) **(3 points)** Find the inverse of the function $g(x)$.

(c) **(2 points)** Write formulas, which need not be simplified, for $(f - g)(x)$ and $\frac{f}{g}(x)$.

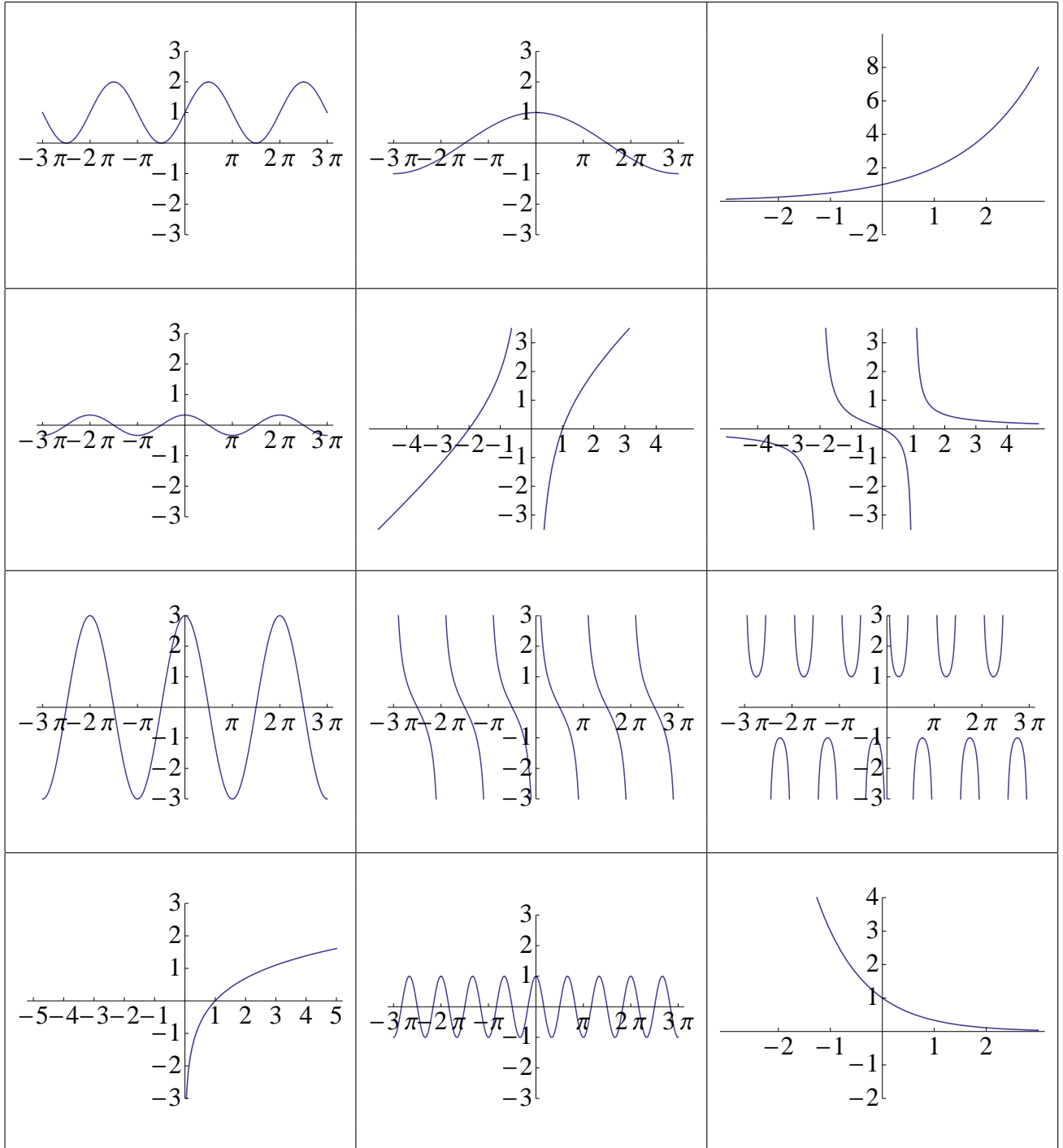
(d) **(4 points)** Determine the domains of $(f - g)(x)$ and $\frac{f}{g}(x)$.

(e) **(3 points)** Write formulas, which need not be simplified, for $f(g(x))$ and $g(g(x))$.

3. (12 points) The following twelve graphs are of the following functions:

$$\begin{array}{llll}
 A(x) = 1 + \sin x & B(x) = \ln x & C(x) = \csc 2x & D(x) = \cot x \\
 E(x) = 3 \cos x & F(x) = \cos \frac{x}{3} & G(x) = \cos(3x) & H(x) = \frac{1}{3} \cos x \\
 I(x) = \left(\frac{1}{3}\right)^x & J(x) = 2^x & K(x) = \frac{x}{(x-1)(x+2)} & L(x) = \frac{(x-1)(x+2)}{x}
 \end{array}$$

Label each picture with the letter of the appropriate function.



4. **(13 points)** Answer the following questions about growth and decay.
- (a) **(3 points)** A certificate of deposit (or CD) bought today will increase in value at a relative growth rate of 1.75% per year. If you buy a \$1000 CD today, how long will it take to reach a value of \$1500?
- (b) **(2 points)** Betaphenethylamine is metabolized and flushed from the bloodstream in such a manner that after one hour, 15% of the drug has been eliminated. Produce a function describing the quantity of the drug still present t hours after administration of a 60mg dose.
- (c) **(3 points)** Betaphenethylamine users suffer visual hallucinations at levels of 25mg or more. Using the function found in the previous part, determine how long it will take after administration of a 60mg dose for the hallucinations to cease.
- (d) **(2 points)** The population of bacteria in a petri dish doubles every 5 hours. If a colony of the bacteria initially consists of 10 cells, produce a function describing the number of bacteria in the colony after t hours.
- (e) **(3 points)** A pot of soup is removed from a hot stove and put in a refrigerator; its temperature in degrees Fahrenheit t minutes after being placed in the fridge is $f(t) = 45 + 150e^{-0.01t}$. What is the original temperature of the soup and the temperature of the refrigerator, and how long will it take to cool to 50°F? Label each of your answers.

5. **(6 points)** Calculate the following trigonometric expressions.

6. **(2 points)** $\arccos \frac{-\sqrt{3}}{2}$.

7. **(2 points)** $\tan \frac{29\pi}{4}$

8. **(2 points)** $\sec \frac{-4\pi}{3}$.

9. **(14 points)** Answer the following questions about sequences and series.

(a) **(4 points)** Identify each of the following sequences as arithmetic, geometric, or neither, and give its common ratio or difference if applicable.

- 1, 4, 9, 16, 25, ...
- 2, 6, 18, 54, 162, ...
- $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}, \dots$
- 2, -5, -12, -19, -26, ...

(b) **(4 points)** The fourth term of an arithmetic sequence is 6 and the ninth term is 31. What is the second term of the sequence?

(c) **(3 points)** What is the sum of the first 50 terms of the arithmetic sequence 4, 2, 0, -2, -4, ...?

(d) **(3 points)** Leaving at most one unsimplified exponent in your answer, evaluate the geometric partial sum $5 - 10 + 20 - 40 + 80 - \dots - 5 \cdot 2^{15}$.

10. (10 points) Answer the following questions about logarithms.

(a) (4 points) Calculate the following logarithms exactly, giving numerical answers:

- $\log_3 81$.
- $\log_5 \frac{1}{25}$
- $\log_6 1$.
- $\log_4 \frac{1}{8}$

(b) (3 points) Calculate the value of the expression $\log_3 21 - \log_3 28 + 2 \log_3 6$ exactly.

(c) (3 points) Find a value of x such that $4 + 2 \log_3 x = 8$.

11. (10 points) Answer the following trigonometric questions.

(a) (3 points) If θ describes a point in quadrant IV and $\sin \theta = \frac{-1}{3}$, what is $\tan \theta$?

(b) (4 points) Simplify the expression $\cot(\arcsin x)$ to a form which does not use trigonometric functions.

(c) (3 points) Find the value of s in the triangle (not drawn to scale) below.

