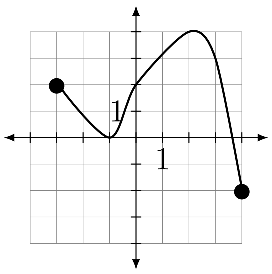
Show work for problems 2 and 5; use the back of the sheet if necessary.

1. **(3 points)** *Giving your answers either in interval form or as inequalities in x, identify the intervals on the following graph on which the function graphed is increasing and the intervals on which it is decreasing. Label which is which.*



By visual inspection, one can see that this decreases on the interval $[-3,-1]$, increases on $[-1,2]$, and decreases again on $[2,3]$.

2. **(4 points)** *Determine the average rate of change of the function $f(z) = 1-3z^2$ between $z=-1$ and $z=2$.*

We calculate the rate of change as a difference of function evalautions, divided by the difference of the two points at which it is being evaluated:

$$\frac{f(2)-f(-1)}{2-(-1)} = \frac{(1-3\cdot 2^2)-(1-3(-1)^2)}{3} = \frac{-9}{3} = -3$$

3. **(6 points)** *Suppose we know what the graph of $f(x)$ looks like. Describe the transformations used to produce the graph of each of the following functions from the graph of $f(x)$.*

   (a) $g(x) = -2f(x)$.
   This is the result of multiplying $f(x)$ by 2, and then inverting it, so the resulting transformation is a vertical stretch by a factor of 2, and a vertical flip (i.e. about the x-axis).

   (b) $h(x) = f(x+2)$.
   This is the result of adding 2 to the argument of $f(x)$, so the resulting transformation is a shift to the left by 2.

   (c) $r(x) = f(\frac{x}{2}) + 1$.
   This is the result of dividing the argument of $f(x)$ by 2 and then adding 1 to the entire expression, which results in a horizontal stretch by a factor of 2, and a shift upwards by 1.

4. **(3 points)** *Consider the quadratic function $f(x) = 2(x-3)^2 - 1$. Where is its vertex, and would its graph open upwards or downwards?*

This quadratic is given in standard form, which already encapsulates the necessary information: specifically, matching $2(x-3)^2-1$ with the template $a(x-h)^2+k$ yields $h=3$, $k=-1$, and $a=2$. Since the vertex of a quadratic in standard form is (h,k), this graph has vertex at $(3,-1)$, and since a is positive, the graph opens upwards.

5. **(4 points)** *Express the quadratic $g(x) = 2x^2+8x+11$ in standard form.*

We perform a standard square-completion:

$$\begin{aligned} g(x) &= 2x^2 + 8x + 11 \\ &= 2(x^2 + 4x) + 11 \\ &= 2(x^2 + 4x + 4 - 4) + 11 \\ &= 2(x^2 + 4x + 4) - 8 + 11 \\ &= 2(x + 2)^2 + 3 \end{aligned}$$