Show work for all problems; use the back of the sheet if necessary.

1. **(3 points)** *Using long division, find the quotient and remainder of the division* $\frac{6x^3+2x^2-22}{x^2-5}$. *Label which is which.*

   Here is the division:

$$
\begin{array}{rrrrr}
 & & & 6x & +2 \\
\cline{2-5}
x^2-5 \big) & 6x^3 & +2x^2 & & -22 \\
 & -6x^3 & & +30x & \\
\cline{2-4}
 & & 2x^2 & +30x & -22 \\
 & & -2x^2 & & +10 \\
\cline{3-5}
 & & & 30x & -12
\end{array}
$$

   Note this uses a slightly different scheme than we learned, in that each row is the negation of what we use and the rows are added rather than subtracted; unfortunately the typesetting package I am using for this purpose (the LaTeX package `polynom.sty`, for the idly curious) doesn't allow flexibility in this particular regard.

   We can now extract the quotient $6x+2$ and the remainder $30x-12$.

2. **(6 points)** *Find all the rational zeroes of the polynomial* x^3-2x^2-7x-4.

   The possible rational roots are ±1, ±2, and ±4. We can go through those quickly by synthetic division:

$$
\begin{array}{c|rrrr}
 & & 1 & -1 & -8 \\
\boxed{1} & 1 & -2 & -7 & -4 \\
\hline
 & 1 & -1 & -8 & -12
\end{array}
\qquad\qquad
\begin{array}{c|rrrr}
 & & -1 & 3 & 4 \\
\boxed{-1} & 1 & -2 & -7 & -4 \\
\hline
 & 1 & -3 & -4 & 0
\end{array}
$$

   so we already found one zero: $(x^3-2x^2-7x-4) = (x+1)(x^2-3x-4)$. The remaining zeroes can be found either by continuing synthetic division on x^2-3x-4 or the quadratic formula; in either case we get the additional zeroes -1 and 4. Thus the full collection of rational zeroes is -1 (with multiplicity 2) and 4.

3. **(4 points)** *Perform the division* $\frac{5-i}{3+4i}$ *and write the result in the form* $a+bi$.

   To divide complex nubmers, we multiply both the numerator and denominator by the complex conjugate of the denominator:

$$\frac{5-i}{3+4i} = \frac{(5-i)(3-4i)}{(3+4i)(3-4i)} = \frac{15-20i-3i-4}{9-12i+12i+16} = \frac{11-23i}{25} = \frac{11}{25} - \frac{23}{25}i$$

4. **(7 points)** *Find the y-intercept, x-intercepts (a.k.a. zeroes), vertical asymptotes, and long-term behavior (a.k.a. nonvertical asymptotes) of the function* $f(x) = \frac{2x}{(x-1)(x-4)}$. *Label which is which.*

   The y intercept is merely $f(0) = 0$. To find zeroes and vertical asymptotes, we start by finding the zeroes of the numerator and denominator of the rational function: $2x$ is zero only when $x = 0$, and $(x-1)(x-4)$ is zero when $x = 1$ or $x = 4$. Thus, $x = 0$ is a zero (or x-intercept) of $f(x)$, and $x = 1$ and $x = 4$ are vertical asymptotes.

   Finally, to explore long term behavior, we may note that for x of very large magnitude:

$$f(x) = \frac{2x}{x^2-5x+4} \approx \frac{2x}{x^2} = \frac{2}{x} \approx 0$$

   so as x gets very large or very small, $f(x)$ tends towards 0.