Show work for all problems; use the back of the sheet if necessary.

1. **(4 points)** *Find a function of the form* $f(x) = Ca^x$ *for a graph which passes through the points* $(0, 4)$ *and* $(3, 108)$.

   We know that the form of the function is $f(x) = Ca^x$; substituting in $x = 0$, we get:

$$\begin{aligned} f(0) &= Ca^0 \\ 4 &= C \end{aligned}$$

   so our form is more specifically $f(x) = 4a^x$. Substituting in $x = 3$, we get:

$$\begin{aligned} f(3) &= 4a^3 \\ 108 &= 4a^3 \\ 27 &= a^3 \\ 3 &= a \end{aligned}$$

   so the function is $f(x) = 4 \cdot 3^x$.

2. **(4 points)** *Evaluate the following logarithms.*

   (a) $\log_{49} 7$.
   Since $7 = \sqrt{49}$, or more relevantly, $7 = 49^{1/2}$, it follows that $\log_{49} 7 = \frac{1}{2}$.

   (b) $\log_2 32$.
   Since $32 = 2^5$, it follows that $\log_2 32 = 5$.

   (c) $\log_6 0$.
   Since there is no x such that $6^x = 0$, this quantity does not exist.

   (d) $\log_5 \frac{1}{25}$.
   Since $\frac{1}{25} = \frac{1}{5^2} = 5^{-2}$, it follows that $\log_5 \frac{1}{25} = -2$.

3. **(4 points)** *Find the domain of the function* $f(x) = \log_5(8 - 2x)$.

   Since the logarithm is only defined for positive arguments, it is necessary that $8 - 2x > 0$, or that $8 > 2x$, so $x < 4$; alternatively, the interval form $(-\infty, 4)$ can be given.

4. **(4 points)** *Calculate the expression* $2\log_3 10 - \log_3 18 - \log_3 50$.

   Using logarithm-simplification rules:

$$\begin{aligned} 2\log_3 10 - \log_3 18 - \log_3 50 &= \log_3 100 - \log_3 18 - \log_3 50 \\ &= \log_3 \frac{100}{18} - \log_3 50 \\ &= \log_3 \frac{100}{18 \cdot 50} \\ &= \log_3 \frac{1}{9} \end{aligned}$$

   and since $\frac{1}{9} = 3^{-2}$, it follows that $\log_3 \frac{1}{9} = -2$.

5. **(4 points)** *Expand the logarithm* $\ln\sqrt{\frac{(x^3y)^3}{\sqrt{z}}}$ *into an expression in terms of* $\ln x$, $\ln y$, *and* $\ln z$

Using logarithm decomposition rules:

$$\begin{aligned}
\ln\sqrt{\frac{(x^3y)^3}{\sqrt{z}}} &= \ln\left(\frac{(x^3y)^3}{\sqrt{z}}\right)^{1/2} \\
&= \frac{1}{2}\ln\frac{(x^3y)^3}{\sqrt{z}} \\
&= \frac{1}{2}\left(\ln(x^3y)^3 - \ln\sqrt{z}\right) \\
&= \frac{1}{2}\left(3\ln(x^3y) - \ln(z^{1/2})\right) \\
&= \frac{1}{2}\left[3\left(\ln(x^3) + \ln y\right) - \frac{1}{2}\ln z\right] \\
&= \frac{1}{2}\left[3\left(3\ln x + \ln y\right) - \frac{1}{2}\ln z\right]
\end{aligned}$$

This can, but need not, be simplified to $\frac{9}{2}\ln x + \frac{3}{2}\ln y - \frac{1}{4}\ln z$.