Show work for all problems; use the back of the sheet if necessary. Exponential and logarithmic expressions need not be simplified, but logarithms must be stated in terms of either natural or common (base 10) logarithms.

1. **(5 points)** *Solve the exponential equation* $(2^{x+1})^2 = 3 \cdot 2^{x+1} + 4$.

   We collect the terms to form a quadratic equation in 2^{x+1}:

   $$\left(2^{x+1}\right)^2 - 3 \cdot 2^{x+1} - 4 = 0$$

   which can be solved by factorization or the quadratic formula; we see the quadratic formula below:

   $$2^{x+1} = \frac{3 \pm \sqrt{9+16}}{2} = \frac{3 \pm 5}{2} = 4 \text{ or } -1$$

   so the solutions to the above equation are exactly the solutions to $2^{x+1} = 4$ and $2^{x+1} = -1$. The latter equation has no solutions, since 2^{x+1} will always be positive; however the former can be solved as such:

   $$\begin{aligned} 2^{x+1} &= 4 \\ x + 1 &= \log_2 4 = 2 \\ x &= 1 \end{aligned}$$

   so $x = 1$ is the only solution to this equation.

2. **(5 points)** *Solve the logarithmic equation* $\log_5 x + \log_5(x+1) = \log_5 20$.

   We gather the logarithms:

   $$\begin{aligned} \log_5 x + \log_5(x+1) &= \log_5 20 \\ \log_5 x + \log_5(x+1) - \log_5 20 &= 0 \\ \log_5[x(x+1)] - \log_5 20 &= 0 \\ \log_5 \frac{x(x+1)}{20} &= 0 \end{aligned}$$

   and now we can recast a logarithmic equation as an exponential one:

   $$\begin{aligned} \frac{x(x+1)}{20} &= 5^0 = 1 \\ x(x+1) &= 20 \\ x^2 + x - 20 &= 0 \end{aligned}$$

   which can be solved either via factorization or the quadratic formula to give the results $x = 4$ and $x = -5$. However, one of these does not correspond to an actual solution: when $x = -5$, the left side of the original equation is nonexistent, so only $x = 4$ is a valid solution to the original equation.

3. **(5 points)** *The population of the town of Risembool is currently 600, and its population is growing at a steady rate of 2% per year. In how many years will the population reach 700?*

A straightforward exponential model for a city with an initial population of 600 which gains 2% to its population over the course of each year is $f(t) = 600(1.02)^t$. We then can solve the desired outcome $f(t) = 700$ for the value of t at which it occurs:

$$\begin{aligned} 700 &= 600(1.02)^t \\ \frac{7}{6} &= 1.02^t \\ t &= \log_{1.02}\frac{7}{6} = \frac{\ln\frac{7}{6}}{\ln 1.02} \end{aligned}$$

which is approximately 7.78 years.

4. **(5 points)** *Find the reference number* $\hat{t}$ *and the terminal point of* $t = \frac{-41}{4}\pi$.

Since $\frac{-41}{4}\pi = -10\pi - \frac{\pi}{4}$, the reference number is $\left|-\frac{\pi}{4}\right| = \frac{\pi}{4}$, so the terminal point is $\left(\pm\frac{\sqrt{2}}{2}, \pm\frac{\sqrt{2}}{2}\right)$. Then, since a rotation of -10π returns to the positive x-axis, and the addition of $-\frac{\pi}{4}$ moves it slightly clockwise, we can be certain that t corresponds to a point in the fourth quadrant, where x-coordinates are positive and y-coordinates negative. Thus, the terminal point is specifically $\left(+\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$.