Show work for all problems except #5; use the back of the sheet if necessary. Final answers on all questions should not contain unsimplified trigonometric expressions.

1. **(3 points)** *Determine the value of* $\sin\left(\frac{\pi}{18}\right)\cos\left(\frac{\pi}{9}\right)+\sin\left(\frac{\pi}{9}\right)\cos\left(\frac{\pi}{18}\right)$.

   We recognize the above expression as a trig-addition formula, so:

$$\sin\left(\frac{\pi}{18}\right)\cos\left(\frac{\pi}{9}\right)+\sin\left(\frac{\pi}{9}\right)\cos\left(\frac{\pi}{18}\right)=\sin\left(\frac{\pi}{18}+\frac{\pi}{9}\right)=\sin\frac{\pi+2\pi}{18}=\sin\frac{\pi}{6}=\frac{1}{2}$$

2. **(3 points)** *Determine the value of* $\cos 75°$.

   We can write 75° as the sum of the two familiar angles 30° and 45°, and then calculate its cosine using angle-addition formulas:

$$\begin{aligned}\cos(75°) &= \cos(30°+45°)\\ &= \cos(30°)\cos(45°)-\sin(30°)\sin(45°)\\ &= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}-\frac{1}{2}\cdot\frac{\sqrt{2}}{2}\\ &= \frac{\sqrt{6}-\sqrt{2}}{4}\end{aligned}$$

3. **(4 points)** *Given that* $\sec x = 3$ *and that* x *is in quadrant IV, calculate* $\cos(2x)$.

   In order to calculate $\cos(2x)=\cos^2 x-\sin^2 x$, we need to know $\sin x$ and $\cos x$. One of these is easy: $\cos x = \frac{1}{\sec x}=\frac{1}{3}$. For the other, we use the Pythagorean identity:

$$\begin{aligned}\cos^2 x+\sin^2 x &= 1\\ \left(\frac{1}{3}\right)^2+\sin^2 x &= 1\\ \sin^2 x &= 1-\frac{1}{9}\\ \sin x &= \pm\sqrt{\frac{8}{9}}\end{aligned}$$

   Using the fact that x is in quadrant IV, we can see that $\sin x = \frac{-\sqrt{8}}{3}$, although this information turns out to be completely unnecessary, because even from simply knowing $\sin^2 x$, we can perform the necessary calculation:

$$\cos(2x)=\cos^2 x-\sin^2 x=\frac{1}{9}-\frac{8}{9}=\frac{-7}{9}$$

   Alternatively, one could avoid working with $\sin x$ at all, by using the cosine double-angle formula $\cos(2x)=2\cos x-1$.

4. **(3 points)** *Determine the value of* $\cos\frac{3\pi}{8}$.

   Using the half-angle formula:

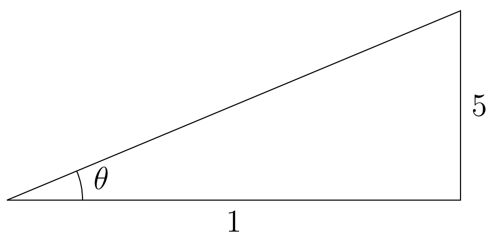
$$\cos\frac{3\pi}{8}=\sqrt{\frac{1+\cos\frac{3\pi}{4}}{2}}=\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}=\frac{\sqrt{2-\sqrt{2}}}{2}$$

5. **(3 points)** *Determine the values of the following inverse trigonometric functions or assert that they do not exist:*

   - $\sin^{-1}\frac{1}{2}$.
     Since $\sin\frac{\pi}{6} = \frac{1}{2}$, and $\frac{\pi}{6}$ is between $\frac{-\pi}{2}$ and $\frac{\pi}{2}$, we know $\sin^{-1}\frac{1}{2} = \frac{\pi}{6}$.
   - $\arccos(-1)$.
     Since $\cos\pi = -1$, and π is between 0 and π (inclusive), we know $\arccos -1 = \pi$.
   - $\arctan(-\sqrt{3})$.
     Since $\tan\frac{-\pi}{3} = -\sqrt{3}$, and $\frac{-\pi}{3}$ is between $\frac{-\pi}{2}$ and $\frac{\pi}{2}$, we know $\arctan(-\sqrt{3}) = \frac{-\pi}{3}$.

6. **(4 points)** *Determine the value of* $\cos\bigl(\tan^{-1}(5)\bigr)$.

   We give a simple name to $\tan^{-1} 5$; traditionally we might call it θ. To convey this information we might build a triangle exemplifying this relationship between θ and 5, which we might write as $\theta = \arctan 5$, but more comprehensibly as $\tan\theta = 5$; in a right triangle with θ as one of the angles, we know that $\tan\theta$ represents the ratio of the lengths of the opposite and adjacent sides. We would thus represent this relationship by making the opposite side of the triangle have length 5, and the asdjacent side have length 1, as shown here:



   Furthermore, the hypotenuse, which is not labeled in the above picture, can be calculated by the Pythagorean Theorem to have length $\sqrt{1^2 + 5^2} = \sqrt{1 + 25} = \sqrt{26}$.

   After that setup, our actual calculation ends up being very easy! We wanted to find $\cos\bigl(\tan^{-1}(5)\bigr)$, which in light of our definition of θ can be written as simply $\cos\theta$. Finding any trigonometric identity of an angle in a labeled right triangle is simply a matter of dividing the lengths of the appropriate sides: the cosine is the ratio of the length of the adjacent side and hypotenuse, so in this case, $\cos\theta = \frac{1}{\sqrt{26}}$.