1 Introduction

Go over the syllabus, noting in particular the assessment dates and the need for class participation.

2 The Language of Math

Chapter 0 gets somewhat ahead of itself on background, but gives some good advice. I would pay particular attention, for stylistic skill, to "Using Symbols #6 (don't mix words and symbols), #7 (don't introduce symbols unless necessary), #8 (identify symbols by type when introduced), #9 (use symbols of standard name according to their function). Mention the symbol-sheet on Blackboard. Also mention language issues like using 1st person plural.

3 Set Definitions

A set is an *unordered* collection of *distinct* objects; these objects can be practically anything, although self-reference in the description of sets is fraught with peril: for now, we'll assume that the elements of a set exclude the set itself. But a set can contain numbers, functions, expressions, and other sets. We will start with some basic notation.

Brackets A set can be explicitly described by listing its elements in brackets, e.g. $\{1, 2, \pi\}$ is a set with the three elements 1, 2, and π, or, more exotically, $\{1, \{\{\}\}, \{2, 4\}, x^2 - 2x + 1\}$, which is a set of four elements, two of which are themselves sets, and one of which is a polynomial.

Ellipses If a pattern is evident, one can use an ellipsis to save space in depicting a finite set, or to give any concept at all of an infinite set. For instance, one might write $\{2, 4, 6, 8, \ldots, 100\}$ to briefly describe the set consisting of the 50 positive even numbers less than or equal to 100, or $\{1, 2, 4, 8, 16, 32, \ldots\}$ to describe the set of powers of 2. If necessary, ellipses can be placed on both ends of an infinite set, e.g. $\{\ldots, -35, -25, -15, -5, 5, 15, 20, 25 \ldots\}$

Conventional names Sets are almost always labelled with italic uppercase Roman letters. If there is only one set being discussed in a context, S or A are the most common names. If there are two, the pairs S, T, X, Y, and A, B are popular. If there are three, A, B, C is a common triple of names. We could, for instance, say "Let $S = \{1, 2, 3\}$ to declare that the name S will henceforth refer to that 3-element set.

The null set The set with no elements whatsoever is called the null set. It can be written $\{\}$ but is conventionally written $\emptyset$.

Inclusion We may assert that a is an element of A with the symbolic notation $a \in A$; likewise, we may say that a is not an element of A with $a \notin A$. For instance, if $S = \{1, 3, 5, 7, 9, \ldots\}$, then we might assert truthfully that $21 \in S$, or that $-5 \notin S$, or $20 \notin S$.

There is another specification which is useful, sometimes, particularly for denoting infinite sets, or sets dependent on an unknown parameter. This specification is of the form: $\{x : p(x)\}$ where x might be any name (usually chosen to be appropriate to its type), and $p(x)$ is some proposition describing it. Here is an example:

$$S = \{n : n \text{is a positive even integer less than } 21\}$$

which describes the same 10-element set as writing $S = \{2, 4, 8, \ldots, 20\}$. However, we can do some interesting things with this notation which would *not* be feasible to do with ellipses, as in this example:

$$T = \{x : x \text{ is a real number and } x < 3\}.$$

which would be unfeasible to list. Note that the list $\{\ldots, -1, 0, 1, 2\}$ does *not* satisfactorily describe this set, since that would suggest a set only containing integers. The propositional description is sufficient, and we can make assertions such as that $e \in T$ since e is a real number less than 3, or $\pi \notin T$, since π is not less than 3, or that $3 + 2i \notin T$ because $3 + 2i$ is not real.

Two notable points on style. First, we need not use this "colon notation"; description in plain English will do as well. We could as easily say "T is the set of all real numbers less than 3" to convey the same idea. Also, it is sometimes not necessary to explicitly claim the type of the variable; the type is often contextually determined, and is a real number if numeric, so the above example could be shortened to $T = \{x : x < 3\}$ and convey the desired information.

Often we will use this colon notation to depict a set as a refinement, or a filter, on another set. So we might take, say $A = \{1, 4, 5, 10, 17\}$ and want to let B consist of the even elements of A (in this case, easily written as $\{4, 10\}$, but on an infinite or arbitrarily-defined set A this may not be as simple). We could say this in words: "Let B consist of the even elements of A" will do, but we could also use the variant of colon notation to say: "Let $B = \{n \in A : n \text{ is even}\}$."

This allows us to succinctly state the context of variables given a few standardized names. The following names are universal! These symbols are always in some variety of bold, either computer bold, or the here-depicted "blackboard bold". The natural numbers (positive integers) are denoted by $\mathbb{N}$; the integers are $\mathbb{Z}$; the rational numbers are denoted by $\mathbb{Q}$; the real numbers are denoted by $\mathbb{R}$. Somewhat less commonly used, the positive rational numbers are $\mathbb{R}^+$, and the complex numbers are $\mathbb{C}$. So now we can give some examples of sets using these context variables:

$$\{n \in \mathbb{N} : n \text{ is a factor of } 12\} = \{1, 2, 3, 4, 6, 12\}$$
$$\{n \in \mathbb{Z} : -3 < n \leq 1\} = \{-2, -1, 0, 1\}$$
$$\{x \in \mathbb{Q} : x^3 - 3x = 0\} = \{0\}$$
$$\{x \in \mathbb{R} : x^2 = -1\} = \emptyset$$