1. **(10 points)** Consider the statements $P : x^2 - 3x + 2 = 0$ and $Q : x \geq 0$.

   (a) **(6 points)** Explain in words why $P \Rightarrow Q$ is true.

   (b) **(4 points)** There are four pairs of truth values for P and Q. For each of the four pairs, either find a value of x corresponding to those truth values, or explain why such an x cannot exist.

2. **(9 points)** The statements $(\neg P) \vee (\neg Q)$ and $\neg(P \wedge Q)$ are equivalent. You will demonstrate this two ways.

   (a) **(4 points)** Fill in the following truth table, and note that the two columns corresponding to $(\neg P) \vee (\neg Q)$ and $\neg(P \wedge Q)$ have identical entries.

P	Q	$\neg P$	$\neg Q$	$(\neg P) \vee (\neg Q)$	$P \wedge Q$	$\neg(P \wedge Q)$
T	T					
T	F					
F	T					
F	F					

   (b) **(5 points)** Write out the statements $(\neg P) \vee (\neg Q)$ and $\neg(P \wedge Q)$ in words instead of symbols, and explain why these two different statements describe the same situation.

3. **(6 points)** Letting P and Q be statements, identify each of the following statements as either tautological, contradictory, or neither. Justify your results, either with explanation or exhaustive computation.

   - $(P \wedge (P \Rightarrow Q)) \Rightarrow Q$.
   - $(P \vee Q) \Leftrightarrow (P \wedge Q)$.
   - $(P \wedge Q) \Rightarrow \neg Q$.

4. **(5 points)** Let P be the proposition "n is a prime number between 4 and 10". For each of the propositions given below, indicate whether that proposition is a necessary condition for P, a sufficient condition for P, both, or neither; briefly justify your claim.

   - Q_1: n is equal to 5.
   - Q_2: n is a positive, odd integer.
   - Q_3: n is a prime number less than 6.

5. **(5 points)** Below, we shall discuss the true statement "For every rational number r, $\frac{1}{r}$ is rational."

   (a) **(2 points)** Write this statement entirely in symbols. Note that we can assert that some x is rational with the statement $x \in \mathbb{Q}$.

   (b) **(3 points)** Write the negation of this statement in words, in as easy-to-comprehend a way as possible. Do not simply wrap the entire expression in the phrase "it is not the case that...". Note that the statement you produce will in fact be false.

6. **(5 points)** Below, we shall discuss the false statement "There is a rational number r such that $r^2 = 2$."

(a) **(2 points)** Write this statement entirely in symbols.

(b) **(3 points)** Write the negation of this statement in words, in as easy-to-comprehend a way as possible. Do not simply wrap the entire expression in the phrase "it is not the case that...". Note that the statement you produce should be true.

7. **(4 point bonus)** Logic is important in designing computers, since the "true" and "false" properties of a statement correspond to the circuit states of having a high signal (usually 5 volts) and a low signal (0 volts). Due to technical restrictions, early computers had only one basic operation, generally called NAND (standing for "not-and") and written with the symbol $\uparrow$. $P \uparrow Q$ was logically equivalent to $\neg(P \wedge Q)$, as shown in the following truth table.

P	Q	$P \uparrow Q$
T	T	F
T	F	T
F	T	T
F	F	T

While this was the only primitive operation which was feasible, early computer scientists of course wanted to use the more familiar and useful operations. Show that one can express the statements $\neg P$, $P \wedge Q$, and $P \vee Q$ entirely in terms of repeated application of the "nand" operation to P and Q in various combinations.