1. **(15 points)** Below are two proofs relating to divisibility of products.

   (a) **(7 points)** Determine the sufficient (and if possible necessary) conditions on an integer n for the following statement to be true: "For integers a and b, if $n \nmid ab$, then $n \nmid a$ and $n \nmid b$.". Then state the condition on n and the resulting impliccation as a proposition and prove it.

   (b) **(8 points)** Determine the sufficient (and if possible necessary) conditions on an integer n for the following statement to be true: "For integers a and b, if $n \mid ab$, then either $n \mid a$ or $n \mid b$.". Then state the condition on n and the resulting impliccation as a proposition and prove it.

2. **(12 points)** In class we saw a proof that $\sqrt{2}$ is irrational. Here we will explore variations on it.

   (a) **(8 points)** Modify the proof of $\sqrt{2}$'s irrationality to produce a proof that for integer n, $\sqrt{n}$ is irrational if n is not exactly the square of some integer.

   (b) **(4 points)** Explain why this proof couldn't be modified to prove that $\sqrt{4}$ is irrational. Don't merely point out that $\sqrt{4}$ is rational; exhibit why the specific proof of the irrationality of $\sqrt{2}$ doesn't work when the 2 is replaced with a 4.

3. **(7 points)** Let $a, b \in \mathbb{Z}$. Prove that if $a^2 + 2b^2 \equiv 0 \pmod{3}$, then either a and b are both congruent to zero modulo 3 or neither is congruent to zero modulo 3.

4. **(6 points)** Let A, B, and C be sets. Prove that $(A - B) \cup (A - C) = A - (B \cap C)$.

5. **(4 point bonus)** Prove (possibly following the structure of the similar proof in class) that if every two vertices of a regular 17-sided polygon are joined with segments colored red, green, or blue, then regardless of how the segments are colored, some three vertices are joined by three edges of the same color.

> Die ganzen Zahlen hat der liebe Gott gemacht; alles andere ist Menschenwerk.
> —attributed to Leopold Kronecker

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