1. **(9 points)** The following questions will explore this slightly obscure relation property.

   Definition 1. A relation R on a set S is *antireflexive* if and only if, for all $a \in S$, $(a, a) \notin R$; in other words, $a \not R\, a$ for all $a \in S$.

   (a) **(2 points)** Demonstrate, either by example or explanation, that there exist relations which are neither reflexive nor antireflexive.

   (b) **(7 points)** Prove that for a set S, if $R \subseteq S \times S$ is an antireflexive, symmetric, and transitive relation, then $R = \emptyset$.

2. **(25 points)** The following proofs concern unions and intersections of relations; if R_1 and R_2 are considered as subsets of $S \times S$, we may take $R_1 \cup R_2$ and $R_1 \cap R_2$ to represent the ordinary operations on these sets.

   (a) **(5 points)** Prove or disprove that, for relations R_1 and R_2 on S, if either R_1 or R_2 is reflexive, then the relation $R_1 \cup R_2$ is reflexive.

   (b) **(5 points)** Prove or disprove that, for relations R_1 and R_2 on S, if both R_1 and R_2 are symmetric, then the relation $R_1 \cup R_2$ is symmetric.

   (c) **(5 points)** Prove or disprove that, for relations R_1 and R_2 on S, if both R_1 and R_2 are transitive, then the relation $R_1 \cup R_2$ is transitive.

   (d) **(5 points)** Prove that for equivalence relations R_1 and R_2 on S, $R_1 \cap R_2$ is an equivalence relation *(note: this is the intersection, whereas the previous questions discussed the union)*.

   (e) **(5 points)** If $x \in S$ and R_1 and R_2 are equivalence relations of S, what is the relationship between the equivalence classes of x with respect to R_1, R_2, and $R_1 \cap R_2$?

3. **(6 points)** Prove or disprove and salvage if possible: for $[a], [b] \in \mathbb{Z}_n$ for a positive integer n, if $[a] \cdot [b] = [0]$, then either $[a] = [0]$ or $[b] = [0]$.

4. **(4 point bonus)** Prove that for a positive integer n, the perfect squares lie in at most $\lceil \frac{n+1}{2} \rceil$ different congruence classes modulo n.

> Ha rossz kedvem van, matematizálok, hogy jó kedvem legyen. Ha jó kedvem van, matematizálok, hogy megmaradjon a jó kedvem. —Alfréd Rényi

due March 25