1. **(24 points)** Demonstrate the existence of bijections between the following pairs of sets; you do not need to explicitly construct the bijection (although in some cases doing so may be the eaisest approach), but you must appeal to some line of argumentation that asserts a bijection exists (e.g. the Cantor-Schröder-Bernstein Theorem).

   (a) **(4 points)** The set $\mathbb{Z}$ and the set of positive even integers $\{2, 4, 8, 16, \ldots\}$.

   (b) **(5 points)** The set $\mathbb{N}$ and the set of quadratic functions with integer coefficients $\{ax^2 + bx + c : a, b, c \in \mathbb{Z}\}$

   (c) **(5 points)** The set $\mathbb{N}$ and the set of all *finite* subsets of $\mathbb{N}$.

   (d) **(5 points)** The set $\mathbb{R}$ and the closed interval $[0, 1]$.

   (e) **(5 points)** The half-open interval $[0, 1)$ and the set $[0, 1) \times [0, 1)$.

2. **(6 points)** A real number is called *transcendental* if it is not a root of any polynomial with integer coefficients. Prove that the set of transcendental numbers is uncountable.

3. **(6 points)** Prove that if S and T are denumerable sets, so is $S \times T$.

4. **(4 points)** Show that given a function S and injective function $f : \mathcal{P}(S) \to \mathbb{N}$, S must be finite.

5. **(4 point bonus)** Let a "description" of a number be a finite string of letters that uniquely determines its value: for instance, "the positive square rrot of two" and "the positive root of the polynomial x squared minus two" are both descriptions for $\sqrt{2}$, and "the ratio of the circumference of a circle to its diameter" is a description for π. Prove that almost all real numbers do not have descriptions.

> Hay un concepto que es el corruptor y el desatinador de los otros. No hablo del mal cuyo limitado imperio es la ética; hablo del infinito.
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> —Jorge Luis Borges, "Avatares de la tortuga"