

1. **(10 points)** Let  $S = \{\emptyset, \{1, 2, 3\}, \{2, 4, 6, 8\}\}$ . For each of the following criteria, either find a set satisfying those conditions, or explain why such a set does not exist.
- Find  $A \in S$  such that  $|A| = 2$ .
  - Find  $B \subseteq S$  such that  $|B| = 2$ .
  - Find  $C \in \mathcal{P}(S)$  such that  $|C| = 2$ .
  - Find  $D \subseteq \mathcal{P}(S)$  such that  $|D| = 2$ .
2. **(10 points)** Find an infinite family of sets  $A_1, A_2, A_3, A_4, \dots$  such that

$$\bigcup_{i=1}^{\infty} A_i = [-2, 2)$$

and

$$\bigcap_{i=1}^{\infty} A_i = (-1, 1].$$

Note that  $[a, b)$  is notation for all real numbers between  $a$  and  $b$  including  $a$  but not including  $b$ .

3. **(10 points)** For statements  $P$ ,  $Q$ , and  $R$ , show that  $((P \rightarrow Q) \wedge (Q \rightarrow R)) \rightarrow (P \rightarrow R)$  is a tautology.
4. **(10 points)** Translate the statement  $\exists A : \mathcal{P}(A) = \emptyset$  into words. Now write, in as easily-understood a form as possible, a negation of this statement in words, and also translate your negation back to symbols. Explain why this negation is true (from which we can determine that the original statement is false).
5. **(5 point bonus)** Find as small a set  $A$  as possible such that  $|A \cap \mathcal{P}(A)| \geq 2$ . Exhibit that  $A$  and  $\mathcal{P}(A)$  have at least two elements in common, and briefly defend your contention that it is the smallest such set.