1. **(8 points)** Prove that for integers a and b, it is the case that $a \equiv 8b \bmod 13$ if and only if $b \equiv 5a \pmod{13}$.

2. **(10 points)** Prove that $3 \mid (2n^2 + 1)$ if and only if $3 \nmid n$.

3. **(12 points)** A subset S of $\mathbb{R}$ is called *multiplicatively closed* if for all $a, b \in S$, it is the case that $ab \in S$ (note that a and b might be the same number). For instance, the following sets are multiplicatively closed: $\mathbb{N}$, $\mathbb{Q}$, $\{1, 2, 4, 8, 16, 32, 64, \ldots\}$; on the other hand $\{-1, 0, 1, 2, 3, 4, \ldots\}$ is not multiplicatively closed, since $-1 \cdot 4 = -4$, which is not in S.

   (a) **(2 points)** Identify six finite multiplicatively closed sets (note that $\emptyset$ is such a set, and can be counted as one of your six).

   (b) **(10 points)** Prove that every finite subset of $\mathbb{R}$ with more than three elements is not multiplicatively closed. (Hint: craft a contradiction argument which divides into two cases based on what happens if the set contains an element x with $0 < |x| < 1$ or with $|x| > 1$).

4. **(10 points)** Let A, B, and C be sets. Prove that if $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then $B = C$.

5. **(5 point bonus)** Consider the following conjecture: for any integers a, b, c, and n, it is the case that $ac \equiv bc \pmod{m}$ if and only if $a \equiv b \mod m$. Note that this statement's converse is already known to us by multiplicativity. Either prove this conjecture or disprove it and suggest a more restricted version which is true.