1. **(15 points)** Let $A = \{\emptyset, 3, 4, \{10, 12\}\}$ and let $B = \{\{3\}, 4, \{10\}\}$. Find sets matching the following descriptions or, where such a set does not exist, explain why.
   (a) $A \cap B$.
   (b) A set $X \subset A$ such that $|X| = 1$.
   (c) A set $Y \in A$ such that $|Y| = 1$.

2. **(12 points)** Identify each of the following statements as a tautology, a contradiction, or neither. Show your work.
   (a) $P \Leftrightarrow (Q \vee \neg P)$.
   (b) $P \Rightarrow (Q \Rightarrow P)$.
   (c) $(P \vee Q) \wedge (\neg P \vee \neg Q)$.

3. **(8 points)** Prove or disprove: for any three sets $A$, $B$, and $C$, it is the case that $(A - B) - C \subseteq A - (B - C)$.

4. **(8 points)** Prove or disprove: for integers $a$ and $b$, if $a^2 \nmid b^2$, then $a \nmid b$.

5. **(9 points)** Prove that the identity $1^2 + 2^2 + 3^2 + 4^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$ holds for all positive integers $n$.

6. **(15 points)** For each of the following relations $R$ on given sets $S$, determine whether each of the reflexive, symmetric, and transitive properties hold. Briefly justify your claims.
   (a) $S = \mathcal{P}(\mathbb{N})$, with $R$ given by the criterion that $A \mathrel{R} B$ if and only if $A \cap B = \emptyset$.
   (b) $S = \{1, 2, 3, 4\}$, with $R = \{(1, 1), (1, 2), (2, 1), (2, 3), (3, 2), (3, 3), (3, 4), (4, 3)\}$.
   (c) $S = \mathbb{N}$, with $R$ given by the criterion that $a \mathrel{R} b$ if and only if $a \leq b^2$.

7. **(18 points)** Suppose that $f : A \to B$ is a function; let $R$ be a relation on $A$ such that for $x, y \in A$, the statement $x \mathrel{R} y$ is true if and only if $f(x) = f(y)$.
   (a) Using no specific knowledge about $f$ except that it is a function, prove that $R$ is an equivalence relation.
   (b) If $f$ is injective, what can you say about the equivalence classes of $R$?
   (c) If $f$ is constant (i.e. there is a $b \in B$ such that for every $a \in A$, $f(a) = b$), what can you say about the equivalence classes of $R$?

8. **(15 points)** Identify each of the following functions with the stated domains and codomains, as injective, surjective, bijective, or none of the above. Briefly justify your claim.
   (a) $f : [-2, 2] \to [0, 2]$ given by $f(x) = |x|$.
   (b) $g : \mathbb{N} \to \mathbb{N}$ given by the rule that $g(n)$ is equal to the largest prime factor of $n$, with the special case that for $n < 2$, $g(n) = n$.
   (c) $h : \mathbb{Z} \to \mathbb{Q}$ given by $h(n) = n$.

> Problems worthy
>     of attack
> prove their worth
>     by hitting back. —Piet Hein, "Problems"