1. **(12 points)** Give examples of sets satisfying the following conditions, or explain why they cannot be met:

   (a) sets A, B, and C such that $A \subseteq B \subsetneq C$.

   (b) sets R, S, and T such that $R \in S$, $S \in T$, and $R \notin T$.

   (c) sets X, Y, and Z such that $X \in Y$ and $Y \subsetneq Z$.

2. **(12 points)** Let $A_i = \{i, i+1, i+2, \ldots, i+100\}$, so that each set A_i has 100 elements which are positive integers. Calculate the results of the following indexed set operations:

   (a) $\bigcap_{i=1}^{100} A_i$.

   (b) $\bigcup_{i=1}^{100} A_i$.

   (c) $\bigcap_{i=1}^{102} A_i$.

   (d) $\bigcup_{i=1}^{\infty} A_i$.

3. **(12 points)** Write out truth tables for each of the following statements (you may write them all in one truth table, if you wish):

   (a) $(P \wedge Q) \to Q$.

   (b) $P \wedge \neg(Q \vee P)$.

   (c) $(\neg P) \leftrightarrow (P \vee \neg Q)$.

4. **(6 points)** Determine the converse of the true statement "If $S \subseteq T$, then $|S| \leq |T|$." Is the converse itself true? Either briefly justify your statement or provide a counterexample.

5. **(6 points)** Write the negation of "There is an element n of S such that $n^2 \in S$" as a quantified statement (using a universal or existential quantifier, either in words or in symbols).

6. **(12 points)** Prove that for an integer n, if $5n^3 \not\equiv 0 \pmod 3$, then $n \not\equiv 0 \pmod 3$.

7. **(12 points)** Let a abd b be nonzero integers. Prove that if $a \mid b$ and $b \mid a$, then either $a = b$ or $a = -b$.