1. **(12 points)** Let $S = \{0, 1, 2, 3\}$. For each of the following descriptions, either produce a set matching the description or explain briefly why such a set doesn't exist.
   (a) A set A of 3 elements, such that $A \subseteq \mathcal{P}(S)$.
   (b) A set B of 3 elements, such that $B \in \mathcal{P}(S)$.
   (c) A set C of 3 elements, such that $C \subseteq S$.
   (d) A set D of 3 elements, such that $D \in S$.

2. **(12 points)** For each positive number i, let A_i be the set $\{x \in \mathbb{R} : 1 + \frac{1}{i} \leq x < 2 + \frac{1}{i}\}$, often simply referred to as the half-open interval $\left[1 + \frac{1}{i}, 2 + \frac{1}{i}\right)$. Calculate the following sets:
   (a) $\bigcap_{i=1}^{3} A_i$.
   (b) $\bigcup_{i=1}^{3} A_i$.
   (c) $\bigcup_{i=1}^{\infty} A_i$.

3. **(12 points)** Write out truth tables for each of the following statements (you may write them all in one truth table, if you wish):
   (a) $(P \vee Q) \Leftrightarrow Q$.
   (b) $P \Rightarrow (P \wedge Q)$.
   (c) $P \vee (\neg P \wedge Q)$.

4. **(6 points)** Write out the contrapositive of the statement "If x and y are even, then xy is even."

5. **(6 points)** Write the negation of "For all $x \in S$, $f(x) = 0$." as a quantified statement (using a universal or existential quantifier, either in words or in symbols).

6. **(12 points)** Prove the following: for integers a and b, if $a \mid b$, then $a^2 \mid b^2$.

7. **(12 points)** Prove the following: for an integer n, if $5n - 11$ is even, then n is odd.