1. **(12 points)** Prove that if x and y are positive real numbers, then $\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$.

2. **(12 points)** Prove that for any positive integer n, it is the case that $1+3+5+\cdots+(2n-1) = n^2$.

3. **(12 points)** Let a_n be defined by the formula $a_1 = 1$, $a_2 = 2$, and $a_n = a_{n-1} + 2a_{n-2}$ for $n \geq 3$. Prove that for every positive integer n, it is the case that $a_n = 2^{n-1}$.

4. **(12 points)** For each of the following relations, determine whether it is reflexive, transitive, and/or symmetric, providing a brief explanation for the properties which hold and a counterexample for properties which do not hold.
   - The relation R_1 on $\mathbb{Z}$ given by $x\ R_1\ y$ iff $|x-y| \leq 2$.
   - The relation R_2 on $\mathbb{Q}$ given by $x\ R_2\ y$ iff x and y have the same denominator when written in lowest terms.

5. **(12 points)** Let R on $\mathbb{R}$ be given by the criterion that $x\ R\ y$ when x and y are either both positive, both negative, or both zero. Prove that R is an equivalence relation.

6. **(12 points)** Identify each of the following functions as injective, surjective, bijective, or none of the above, with a brief explanation (or counterexample if appropriate).
   - $f : \mathbb{Z} \to Z$ given by $f(n) = 2n + 1$.
   - $g : \mathbb{R} \to R$ given by $g(n) = 2n + 1$.
   - $h : \mathbb{N} \to N$ given by letting $h(n)$ be the largest value k such that $k \mid n$ but $k \neq n$; for example, $h(105) = 35$, since 35 is the largest nubmer less than 105 which divides 105.