

This test is closed-book and closed-notes. No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified.

1. **(16 points)** Determine the domains of the following functions:

(a) **(4 points)** $f(x) = x^3 - 3x + \frac{1}{x}$.

(b) **(4 points)** $g(t) = \sqrt{9 - t^2}$.

(c) **(4 points)** $h(r) = \frac{\ln(r+4)}{r-3}$.

(d) **(4 points)** $q(s) = \sqrt{8s + 3} - \ln(9 - s)$.

2. **(9 points)** Let $f(x) = 5x - 2$.

(a) **(1 point)** Find $\lim_{x \rightarrow 3} f(x)$.

(b) **(8 points)** Using epsilon-delta methods, justify your result above.

1	/ 16
2	/ 9
3	/ 10
4	/ 13
5	/ 9
6	/ 20
7	/ 7
8	/ 16
9	/ (6)
Σ	/100

3. **(10 points)** Given the function $f(x) = \frac{8-x^3}{x+2}$, answer the following questions preparatory to sketching the functions.

(a) **(3 points)** What is the domain of the function?

(b) **(7 points)** Describe, either in words or symbolically, the long-term behavior of the function in each direction.

4. **(13 points)** Let $f(x) = 20 - 4x - x^2$.

(a) **(9 points)** *Using the difference quotient*, determine the formula for $f'(x)$.

(b) **(4 points)** Find the equation of the tangent line to $f(x)$ at the point $(3, -1)$.

5. (9 points) Let $f(x) = \begin{cases} 3x & \text{if } x < 2 \\ x^2 + a & \text{if } 2 \leq x < 8. \\ b \log_2 x & \text{if } x \geq 8 \end{cases}$.

What choices of a and b will make this function continuous?

6. (20 points) Evaluate the following limits; when a limit can not be evaluated, explicitly say so.

(a) (4 points) $\lim_{q \rightarrow -\infty} \frac{2q^3 + q^2 - 4}{q^5 - 3q}$.

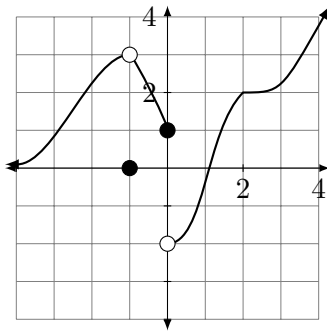
(b) (4 points) $\lim_{\theta \rightarrow 0} \frac{\sin \theta - 2}{\cos \theta + 3 \sec \theta}$.

(c) (4 points) $\lim_{t \rightarrow 4} \frac{t^2 - 8t + 16}{t^2 + t - 20}$.

(d) (4 points) $\lim_{x \rightarrow \infty} \frac{8x^4 - 7x^2 + 2}{6 - 2x^4}$.

(e) (4 points) $\lim_{s \rightarrow 2} \frac{s^2 + 2s - 1}{s^3 - 4s}$.

7. (7 points) For the plot of $h(x)$ shown below, indicate whether or not each of the following quantities can be evaluated. If they can be evaluated, compute their values. If they cannot be evaluated, explicitly say so. You need not show work.



$$\lim_{x \rightarrow -1^-} h(x)$$

$$h(0)$$

$$\lim_{x \rightarrow -1^+} h(x)$$

$$\lim_{x \rightarrow 2} h(x)$$

$$h(-1)$$

$$h(2)$$

$$\lim_{x \rightarrow 0} h(x)$$

8. (16 points) Calculate the following derivatives, using any method you wish.

(a) (4 points) Find $\frac{d}{dt} ((t^3 + 2t)(e^t - 4))$.

(b) (4 points) If $y = 3\sqrt[3]{x} - 8x^5 + \frac{4}{x^3} - 20$, find $\frac{dy}{dx}$.

(c) (4 points) Determine $\frac{d}{dx} \frac{x^2 + 3e^x}{2x + 1}$.

(d) (4 points) For $f(x) = 3x^2 + 4x - 8e^x + \frac{1}{x}$, calculate its second derivative $f''(x)$.

9. (6 point bonus) Find a general formula (in terms of n) for $\frac{d^n}{dx^n} (x^2 e^x)$.