

For full credit show all of your work (legibly!), unless otherwise specified. Answers may include all arithmetic operations, trigonometric functions, inverse trigonometric functions, and natural logarithms. Algebraic simplification of answers is unnecessary.

The problems are in no particular order, and it is suggested that you look at all of them before beginning to answer any.

1. **(12 points)** A bar of iron which has been heated to  $1400^\circ\text{F}$  is taken from the furnace into a  $100^\circ\text{F}$  metalworking studio. After 5 minutes it has cooled to  $800^\circ\text{F}$ .

(a) **(5 points)** Produce a function  $T(t)$  modeling the bar's temperature  $t$  minutes after removal from the furnace.

(b) **(3 points)** How quickly is the bar's temperature changing immediately upon removal from the furnace?

(c) **(4 points)** The metal can be worked as long as it is hotter than  $1000^\circ\text{F}$ . How soon after the bar is removed from the furnace does it become unworkable?

2. **(10 points)** If  $f(r) = \sin(2r^3 \ln r)$ , calculate  $f'(r)$ .

1	/ 12
2	/ 10
3	/ 8
4	/ 15
5	/ 10
6	/ 15
7	/ 20
8	/ 10
9	/ (6)
$\Sigma$	/100

3. **(8 points)** Estimate the following values using appropriate linear approximations.

(a) **(4 points)**  $\sqrt[3]{7.994}$ .

(b) **(4 points)**  $(-2.004)^5$ .

4. **(15 points)** Hiro is motorcycling west at 60 mph from a point 120 miles east of the Black Sun, while Raven is 50 miles north of the Black Sun, driving northwards at 30 mph.

(a) **(12 points)** Are they growing closer together or further apart, and at what speed are they doing so?

(b) **(3 points)** In an hour, will they be growing closer together or further apart, and at what speed will they be doing so?

5. **(10 points)** Find  $\frac{d}{dt} \frac{e^{7t-3}}{\arctan(t^2)}$ .

6. **(15 points)** The *conchoid of de Sluze* is a curve satisfying the equation  $(x-1)(x^2+y^2) = 4x^2$ .

(a) **(12 points)** Find a formula for  $\frac{dy}{dx}$  on this curve.

(b) **(3 points)** Find the equation of the tangent line to the curve at  $(3, 3)$ .

7. **(20 points)** Answer the following derivative-related questions.

(a) **(6 points)** If  $f(x) = (x^2 - 4x) \arcsin(x)$ , find  $f'(x)$ .

(b) **(8 points)** Find  $\frac{d}{dt} (\ln(\cos t))^5$ .

(c) **(6 points)** If  $y = \frac{e^x}{x^3 - \sqrt{x}}$ , find  $\frac{dy}{dx}$ .

8. **(10 points)** Find an equation of the tangent line to the curve  $y = \frac{x^2+3}{x^2+x+1}$  at  $(2, 1)$ .

9. **(6 point bonus)** Complete these two questions on the back of this page.

(a) **(3 point bonus)** Prove the quotient rule using any combination of the other rules (the product rule, chain rule, and derivatives of known functions).

(b) **(3 point bonus)** Prove the product rule using only the chain rule and the derivatives of known functions.