No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified. Algebraic and trigonometric simplification of answers is generally unnecessary.

1. **(24 points)** Answer the following questions related to the shape of the graph of the function $g(x) = x^4 - 8x^2 + 8$.

   (a) **(4 points)** What is $g(x)$'s long term behavior as $x$ grows very large or very negative? Describe each direction in either words or symbols.

   (b) **(6 points)** Where is $g(x)$ increasing? Where is it decreasing? Label which is which.

   (c) **(6 points)** What are its critical points, and is each a local maximum, a local minimum, or neither?

   (d) **(8 points)** Where is it concave up? Where is it concave down? Label which is which. Where, if anywhere, are its points of inflection?

2. **(24 points)** You have 150 square inches of paper with which to design a rectangular poster. The top margin of the poster will be 2 inches, and the bottom, left, and right margins will be 1 inch. What dimensions for the poster maximize the *printable* area?

| | |
|---|---|
| 1 | / 24 |
| 2 | / 24 |
| 3 | / 18 |
| 4 | / 12 |
| 5 | / 22 |
| 6 | / (5) |
| Σ | /100 |

3. **(18 points)** Answer the following questions:

   (a) **(5 points)** Determine a region whose area is $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{5}{n}\right) \arctan(2+\frac{5i}{n})$. You may express your answer as a definite integral, or as a description in words.

   (b) **(7 points)** Find $f(x)$ given that $f'(x) = 16x^3 - 3x^2$ and $f(1) = 4$.

   (c) **(8 points)** Find the general antiderivative of $h(t) = \sqrt[6]{t} + \frac{5}{t} - 2 + 4\csc^2 t - \frac{5}{1+t^2}$.

4. **(12 points)** Answer the following questions about approximation:

   (a) **(6 points)** Starting with an initial value of 1, use two iterations of Newton's method to approximate a zero of $f(x) = x^6 - 5x + 3$. Your answer need not be arithmetically simplified.

   (b) **(6 points)** Choose $x_1 = 4$ to be an initial approximation of $\sqrt{17}$. Use one step of Newton's method on an appropriately chosen polynomial function to develop $x_2$, a better rational approximation of $\sqrt{17}$; also give an arithmetic expression (which need not be simplified) for the better approximation $x_3$ arising from a second step of Newton's method.

5. **(22 points)** Evaluate the following limits; if they cannot be evaluated, show why not.

   (a) $\lim_{x\to 0} \frac{6x}{\arctan x}$.

   (b) $\lim_{\theta\to 0} \frac{\theta+\sin\theta}{\theta+\cos\theta}$.

   (c) $\lim_{u\to +\infty} \frac{e^{u/10}}{u^3}$.

   (d) $\lim_{x\to 0} \frac{\sin x - x}{x^2 e^x}$.

   (e) $\lim_{x\to 1} \frac{\ln x}{x^2-2x+1}$.

6. **(5 point bonus)** We have seen two procedures for estimating a difficult-to-calculate $\sqrt[n]{k}$: we can either choose $a$ close to $\sqrt[n]{k}$ and perform a linear approximation, or choose a reasonable guess $x_0$ and perform Newton's method on an easy-to-manage polynomial which has $\sqrt[n]{k}$ as a zero. Prove on the back of this sheet that, in general, if $x_0 = a$, a single step of Newton's method gives the exact same result as the linear approximation.