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This test is closed-book and closed-notes. No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified. For the purposes of this exam, all answers must be in terms of familiar functions. Algebraic and trigonometric simplification of answers is generally unnecessary.

1. **(12 points)** The *keratoid cusp* is a curve satisfying the equation $y^2 = x^2y + x^5$.

(a) **(9 points)** Find a formula for $\frac{dy}{dx}$ on this curve.

(b) **(3 points)** Find the equation of the tangent line to the keratoid cusp at the point $(2, -4)$.

2. **(12 points)** You are constructing a subdivided rectangular pasture by fencing around the entire pasture, and then partitioning it with one fence running parallel to one pair of edges and two fences parallel to the other pair of edges, dividing the pasture into 6 sections. If the entire pasture is to have an area of 1200 square feet, what is the minimum amount of fencing you can use to build this pasture?

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3. **(12 points)** G-23 paxilon hydrochlorate is eliminated from the bloodstream at a rate of 12% per hour. Miranda has just taken a 40mg intravenous dose.

(a) **(4 points)** Construct a function modeling the quantity of the drug in her body after t hours.

(b) **(4 points)** How quickly is the drug being eliminated after 2 hours?

(c) **(4 points)** How many hours does it take for the quantity of the drug to drop below 10mg?

4. **(12 points)** Let $h(x) = (x^2 + 1)e^x$.

(a) **(5 points)** Where is $h(x)$ increasing? Where is it decreasing?

(b) **(7 points)** Determine $h(x)$'s concavity and identify points of inflection.

5. **(8 points)** Answer the following questions about the function $g(x) = \sqrt{25 - x^2}$.
- (a) **(4 points)** What is the domain of $g(x)$?
- (b) **(4 points)** Where does the derivative of $g(x)$ exist?
6. **(12 points)** Consider the function $g(x) = \frac{e^x}{x+2}$.
- (a) **(5 points)** Identify the zeroes, vertical asymptotes, and long-term behavior on both sides of this function. Clearly label which is which.
- (b) **(5 points)** Identify the critical points of this function, and indicate whether each is a local maximum, local minimum, or neither.
- (c) **(2 points)** Which if any of the critical points identified above are global maxima or global minima? Show work or explain.

7. (12 points) Evaluate the following integrals:

(a) (4 points) $\int_0^2 \frac{\sec \sqrt{x} \tan \sqrt{x}}{\sqrt{x}} dx$

(b) (4 points) $\int_{-2}^1 2x^2 + \frac{3}{x^2+1} dx$

(c) (4 points) $\int \frac{1}{x \ln x} dx$

8. (12 points) Determine the following limits.

(a) (4 points) Evaluate $\lim_{x \rightarrow 0} (e^x - 1) \csc x$ or demonstrate that it cannot be evaluated.

(b) (4 points) *Using the difference quotient*, find the derivative with respect to x of $f(x) = 4x^2 - x + 5$. You may not use L'Hôpital's rule for this problem.

(c) (4 points) Evaluate $\lim_{t \rightarrow \infty} \frac{1}{t^2 e^t}$ or demonstrate that it cannot be evaluated.

9. **(12 points)** A sentry at Blackgate Prison has turned a spotlight on an escapee who is currently 0.3 miles to the north and 0.4 miles to the east of the prison. She notices that the escapee is traveling eastwards at four miles per hour.

(a) **(6 points)** How quickly will she need to rotate the spotlight to keep it trained on the escapee?

(b) **(6 points)** How quickly is the escapee's distance from the prison changing?

10. **(12 points)** Answer the following questions:

(a) **(4 points)** Find $\int \frac{d}{dt} \frac{e^t}{t} dt$.

(b) **(4 points)** Find $\frac{d}{dx} \arctan \frac{x^2-1}{x+2}$.

(c) **(4 points)** Given $g(s) = \frac{d}{ds} e^s \cot(s^2)$, find $g'(s)$.