

1. **(7 points)** *Identify the domains of the following functions:*

(a) **(4 points)**  $g(t) = \sqrt{6 - 2t} + \ln(2 + t)$ .

There are two problematic possibilities: the argument of a square-root function cannot be negative, and the argument of a logarithm cannot be non-positive. Thus, in order for this function to be evaluated, it must be the case that both  $6 - 2t \geq 0$  and  $2 + t > 0$ . These can be algebraically simplified to  $t \leq 3$  and  $t > -2$  respectively, so the domain consists of  $-2 < t \leq 3$ , or, in interval notation,  $(-2, 3]$ .

(b) **(3 points)**  $f(x) = \frac{4x-16}{x^2+x-6}$ .

There is a problematic expression: the denominator of a fraction cannot be zero. Thus, in order for this function to be evaluated, it must be the case that  $x^2 + x - 6 \neq 0$ . This can be algebraically simplified, using factorization or the quadratic formula, to the requirement that  $x \neq -3$  and  $x \neq 2$ , so the domain consists of all  $x$  not equal to  $-3$  or  $2$ , or, in interval notation,  $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$ .

2. **(3 points)** *Find the equation of the line through the points  $(-1, 3)$  and  $(3, 15)$ .*

The slope of this line is given by the formula  $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{15 - 3}{3 - (-1)} = \frac{12}{4} = 3$ . Then its formula can either be put in point-slope form as

$$(y - 15) = 3(x - 3)$$

or one can “plug in” values to slope-intercept form to find the  $y$ -intercept:

$$\begin{aligned} y &= 3x + b \\ 3 &= 3(-1) + b \\ 6 &= b \end{aligned}$$

so the equation of the line will be  $y = 3x + 6$ , or any algebraically equivalent equation.

3. **(4 points)** *This table indicates the position of a runner in the first 5 seconds of a race:*

<i>Time elapsed (in seconds)</i>	<i>0.0</i>	<i>1.0</i>	<i>2.0</i>	<i>3.0</i>	<i>4.0</i>	<i>5.0</i>
<i>Distance traveled (in meters)</i>	<i>0.0</i>	<i>1.2</i>	<i>5.4</i>	<i>11.6</i>	<i>16.2</i>	<i>26.0</i>

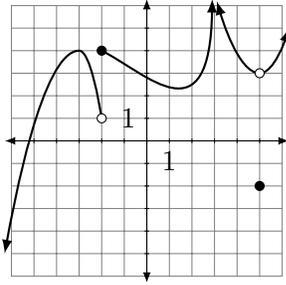
(a) **(2 points)** *What is the runner’s average speed in the first two seconds of the race?*

The average speed of an object with position function  $f(t)$  between two times  $a$  and  $b$  is known to be  $\frac{f(b) - f(a)}{b - a}$ ; here we are measuring the average speed between the start time (0 seconds in) and 2 seconds in, so the average speed is  $\frac{f(2) - f(0)}{2 - 0} = \frac{5.4 - 0.0}{2 - 0} = 2.7$  meters per second.

(b) **(2 points)** *What is the runner’s average speed between the times  $t = 1$  and  $t = 4$ ?*

We proceed as above, but here we are measuring the average speed between 1 second into the race and 4 seconds in, so the average speed is  $\frac{f(4) - f(1)}{4 - 1} = \frac{16.2 - 1.2}{4 - 1} = 5.0$  meters per second.

4. **(6 pts)** *Below is the graph of a function  $f(x)$ . For each of the six quantities listed to the right, give its value if it has a value, or specifically state that it does not exist.*



$f(-2)$  is 4, as evidenced by the solid dot on the graph at  $(-2, 4)$ .

$\lim_{x \rightarrow -2^+} f(x)$  is 4, since the curve slightly to the right of the  $x$ -value  $-2$  is very close to height 4.

$\lim_{x \rightarrow -2^-} f(x)$  is 1, since the curve slightly to the left of the  $x$ -value  $-2$  is very close to height 1.

$\lim_{x \rightarrow -3} f(x)$  is 4, since at  $x$ -values close to  $-3$  (and, in fact, at  $-3$  itself, although this is emphatically not relevant to the question) the curve is very close to height 4.

$\lim_{x \rightarrow 5} f(x)$  is 3, since at  $x$ -values close to 5 (although not at  $x = 5$  itself) the curve is very close to height 3.

$\lim_{x \rightarrow 3} f(x)$  does not exist, since at  $x$ -values close to 3 the curve shoots upwards instead of tending towards a specific value. The idiomatic expression  $\lim_{x \rightarrow 3} f(x) = +\infty$  is often used to describe this behavior, but in keeping with the question asked, it should be specifically stated that this limit does not exist.

5. **(2 point bonus)** *If for every value of  $x$  it is the case that  $f(-x) = -f(x)$  and  $g(-x) = g(x)$ , what (if anything) can be said about  $f(f(x))$ ,  $f(g(x))$ ,  $g(f(x))$ , and  $g(g(x))$ ?*

These conditions are what is sometimes called “function parity”: the description of  $f(x)$  is what is known as an “odd” function, and the description of  $g(x)$  is an “even” function. The functions  $f(f(x))$ ,  $f(g(x))$ ,  $g(f(x))$ , and  $g(g(x))$  also possess parity, as can be seen below:

$$f(f(-x)) = f(-f(x)) = -f(f(x))$$

$$f(g(-x)) = f(g(x))$$

$$g(f(-x)) = g(-f(x)) = g(f(x))$$

$$g(g(-x)) = g(g(x))$$

so  $f(f(x))$  is odd, but the other three are even.