

1. **(7 points)** *Identify the domains of the following functions:*

(a) **(4 points)** $g(t) = \sqrt{6 - 2t} + \ln(2 + t)$.

There are two problematic possibilities: the argument of a square-root function cannot be negative, and the argument of a logarithm cannot be non-positive. Thus, in order for this function to be evaluated, it must be the case that both $6 - 2t \geq 0$ and $2 + t > 0$. These can be algebraically simplified to $t \leq 3$ and $t > -2$ respectively, so the domain consists of $-2 < t \leq 3$, or, in interval notation, $(-2, 3]$.

(b) **(3 points)** $f(x) = \frac{4x-16}{x^2+x-6}$.

There is a problematic expression: the denominator of a fraction cannot be zero. Thus, in order for this function to be evaluated, it must be the case that $x^2 + x - 6 \neq 0$. This can be algebraically simplified, using factorization or the quadratic formula, to the requirement that $x \neq -3$ and $x \neq 2$, so the domain consists of all x not equal to -3 or 2 , or, in interval notation, $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$.

2. **(3 points)** *Find the equation of the line through the points $(-1, 3)$ and $(3, 15)$.*

The slope of this line is given by the formula $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{15 - 3}{3 - (-1)} = \frac{12}{4} = 3$. Then its formula can either be put in point-slope form as

$$(y - 15) = 3(x - 3)$$

or one can “plug in” values to slope-intercept form to find the y -intercept:

$$\begin{aligned} y &= 3x + b \\ 3 &= 3(-1) + b \\ 6 &= b \end{aligned}$$

so the equation of the line will be $y = 3x + 6$, or any algebraically equivalent equation.

3. **(4 points)** *This table indicates the position of a runner in the first 5 seconds of a race:*

<i>Time elapsed (in seconds)</i>	<i>0.0</i>	<i>1.0</i>	<i>2.0</i>	<i>3.0</i>	<i>4.0</i>	<i>5.0</i>
<i>Distance traveled (in meters)</i>	<i>0.0</i>	<i>1.2</i>	<i>5.4</i>	<i>11.6</i>	<i>16.2</i>	<i>26.0</i>

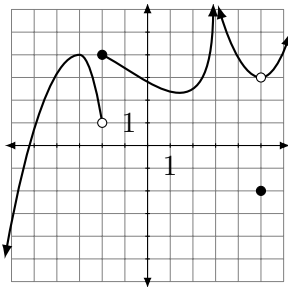
(a) **(2 points)** *What is the runner’s average speed in the first two seconds of the race?*

The average speed of an object with position function $f(t)$ between two times a and b is known to be $\frac{f(b) - f(a)}{b - a}$; here we are measuring the average speed between the start time (0 seconds in) and 2 seconds in, so the average speed is $\frac{f(2) - f(0)}{2 - 0} = \frac{5.4 - 0.0}{2 - 0} = 2.7$ meters per second.

(b) **(2 points)** *What is the runner’s average speed between the times $t = 1$ and $t = 4$?*

We proceed as above, but here we are measuring the average speed between 1 second into the race and 4 seconds in, so the average speed is $\frac{f(4) - f(1)}{4 - 1} = \frac{16.2 - 1.2}{4 - 1} = 5.0$ meters per second.

4. **(6 pts)** *Below is the graph of a function $f(x)$. For each of the six quantities listed to the right, give its value if it has a value, or specifically state that it does not exist.*



$f(-2)$ is 4, as evidenced by the solid dot on the graph at $(-2, 4)$.

$\lim_{x \rightarrow -2^+} f(x)$ is 4, since the curve slightly to the right of the x -value -2 is very close to height 4.

$\lim_{x \rightarrow -2^-} f(x)$ is 1, since the curve slightly to the left of the x -value -2 is very close to height 1.

$\lim_{x \rightarrow -3} f(x)$ is 4, since at x -values close to -3 (and, in fact, at -3 itself, although this is emphatically not relevant to the question) the curve is very close to height 4.

$\lim_{x \rightarrow 5} f(x)$ is 3, since at x -values close to 5 (although not at $x = 5$ itself) the curve is very close to height 3.

$\lim_{x \rightarrow 3} f(x)$ does not exist, since at x -values close to 3 the curve shoots upwards instead of tending towards a specific value. The idiomatic expression $\lim_{x \rightarrow 3} f(x) = +\infty$ is often used to describe this behavior, but in keeping with the question asked, it should be specifically stated that this limit does not exist.

5. **(2 point bonus)** *If for every value of x it is the case that $f(-x) = -f(x)$ and $g(-x) = g(x)$, what (if anything) can be said about $f(f(x))$, $f(g(x))$, $g(f(x))$, and $g(g(x))$?*

These conditions are what is sometimes called “function parity”: the description of $f(x)$ is what is known as an “odd” function, and the description of $g(x)$ is an “even” function. The functions $f(f(x))$, $f(g(x))$, $g(f(x))$, and $g(g(x))$ also possess parity, as can be seen below:

$$f(f(-x)) = f(-f(x)) = -f(f(x))$$

$$f(g(-x)) = f(g(x))$$

$$g(f(-x)) = g(-f(x)) = g(f(x))$$

$$g(g(-x)) = g(g(x))$$

so $f(f(x))$ is odd, but the other three are even.