

Show work for all answers.

1. **(9 points)** Calculate the following limits, or demonstrate that they do not exist:

(a) **(3 points)**  $\lim_{x \rightarrow 4} \frac{x^2 - 4x}{x^2 - 3x - 4}$

(b) **(3 points)**  $\lim_{t \rightarrow 2} \frac{t^4 - 2}{2t^2 - 3t + 2}$

(c) **(3 points)**  $\lim_{x \rightarrow +\infty} \frac{1 - x^2}{x^3 - x + 1}$

2. **(3 points)** Find a value  $a$  so that the function  $f(x) = \begin{cases} 2^x & \text{if } x < 3 \\ ax^2 & \text{if } x \geq 3 \end{cases}$  is continuous everywhere.

3. **(4 points)** Prove by epsilon-delta methods that  $\lim_{t \rightarrow 6} 8 - 3t = -10$ .

4. **(4 points)** Using the difference quotient, find the slope of the tangent line to the graph of  $f(x) = 2x^2 - 3x + 1$  at the point  $(2, 3)$ .

5. **(2 point bonus)** Prove that there are infinitely many values of  $x$  such that  $\tan x = x$ .