

1. (3 points) Calculate $\frac{d}{ds} (2s^7 - \frac{4}{s^5} + 3e^s)$.

We rephrase $\frac{4}{s^5}$ as $4s^{-5}$, and then invoke the power rule and exponential rule:

$$\frac{d}{ds} (2s^7 - 4s^{-5} + 3e^s) = 14s^6 + 20s^{-6} + 3e^s$$

2. (5 points) If $y = 3 \sec x + 2$, calculate its second derivative $\frac{d^2y}{dx^2}$.

First we calculate the derivative $\frac{dy}{dx} = 3 \sec x \tan x$; then, to find the second derivative, we will invoke the product rule, since $\frac{dy}{dx}$ is a product:

$$\begin{aligned} \frac{d^2y}{dx^2} &= \frac{d}{dx} (3 \sec x \tan x) \\ &= \left(\frac{d}{dx} 3 \sec x \right) \tan x + 3 \sec x \frac{d}{dx} \tan x \\ &= 3 \sec x \tan x \tan x + 3 \sec x \sec^2 x = 3 \sec x \tan^2 x + 3 \sec^3 x \end{aligned}$$

3. (4 points) If $f(t) = \frac{t^3 + e^t}{t^2 - 3t}$, calculate $f'(t)$.

This is an application of the quotient rule:

$$\begin{aligned} f'(t) &= \frac{d}{dt} \frac{t^3 + e^t}{t^2 - 3t} \\ &= \frac{(t^2 - 3t) \frac{d}{dt} (t^3 + e^t) + (t^3 + e^t) \frac{d}{dt} (t^2 - 3t)}{(t^2 - 3t)^2} \\ &= \frac{(t^2 - 3t)(3t^2 + e^t) + (t^3 + e^t)(2t - 3)}{(t^2 - 3t)^2} \end{aligned}$$

4. (4 points) Calculate $\frac{d}{dx} \cos(x^3 - 2x)$.

This is an application of the chain rule. If we denote $x^3 - 2x$ as u , then:

$$\begin{aligned} \frac{d}{dx} \cos(x^3 - 2x) &= \frac{d}{dx} \cos(u) \\ &= \frac{du}{dx} \frac{d}{du} \cos(u) \\ &= \frac{d}{dx} (x^3 - 2x) (-\sin u) \\ &= -(x^2 - 2) \sin(x^3 - 2x) \end{aligned}$$

5. (4 points) Find the equation of the tangent line to the curve $y = (2x^2 - 7x + 5)e^x$ at $(0, 5)$.

Using the product rule, we can determine that

$$\frac{dy}{dx} = \left(\frac{d}{dx} (2x^2 - 7x + 5) \right) e^x + (2x^2 - 7x + 5) \frac{d}{dx} e^x = (4x - 7)e^x + (2x^2 - 7x + 5)e^x$$

so when x is 0, $\frac{dy}{dx} = -7e^0 + 5e^0 = -2$. Thus, the tangent line to this curve at $(0, 5)$ has slope of -2 ; since it passes through $(0, 5)$, its equation can then be easily seen to be $y = -2x + 5$.