1. **(6 points)** *Given that $xe^y = x - y$, find $\frac{dy}{dx}$ by implicit differentiation.*

   Let us perform the derivative with respect to x of each side, using the product and chain rules as necessary:

$$\begin{aligned}
\frac{d}{dx}(xe^y) &= \frac{d}{dx}(x-y) \\
\left(\frac{d}{dx}x\right)e^y + x\left(\frac{d}{dx}e^y\right) &= 1 - \frac{dy}{dx} \\
1e^y + x\left(\frac{dy}{dx}\frac{d}{dy}e^y\right) &= 1 - \frac{dy}{dx} \\
e^y + x\frac{dy}{dx}e^y &= 1 - \frac{dy}{dx} \\
(1+x)\frac{dy}{dx}e^y &= 1 - e^y \\
\frac{dy}{dx}e^y &= \frac{1-e^y}{1+x}
\end{aligned}$$

2. **(4 points)** *Calculate $\frac{d}{dt}\frac{\arcsin t}{t^2-1}$.*

   We will use the quotient rule to solve this problem, making use of our new knowledge of the derivitive of the arcsine function.

$$\frac{d}{dt}\frac{\arcsin t}{t^2-1} = \frac{(t^2-1)\frac{d}{dt}\arcsin t - \arcsin t\frac{d}{dt}(t^2-1)}{(t^2-1)^2} = \frac{(t^2-1)\frac{1}{\sqrt{1-t^2}} - 2t\arcsin t}{(t^2-1)^2}$$

3. **(5 points)** *If $f(x) = (\arctan x)\ln(7x^2-2)$, determine $f'(x)$.*

   We will use the product and chain rules to solve this problem, making use of the auxiliary variable $u = 7x^2 - 2$ and our new knowledge of the derivitive of the arctangent and logarithm functions.

$$\begin{aligned}
f'(x) &= \frac{d}{dx}[(\arctan x)\ln(7x^2-2)] \\
&= \left(\frac{d}{dx}\arctan x\right)\ln(7x^2-2) + \arctan x\frac{d}{dx}\ln(u) \\
&= \frac{1}{1+x^2}\ln(7x^2-2) + \arctan x\frac{du}{dx}\frac{d}{du}\ln(u) \\
&= \frac{\ln(7x^2-2)}{1+x^2} + \arctan x(14x)\frac{1}{u} \\
&= \frac{\ln(7x^2-2)}{1+x^2} + \frac{14x\arctan x}{7x^2-2}
\end{aligned}$$

$$\frac{d}{dt}\frac{\arcsin t}{t^2-1} = \frac{(t^2-1)\frac{d}{dt}\arcsin t - \arcsin t\frac{d}{dt}(t^2-1)}{(t^2-1)^2} = \frac{(t^2-1)\frac{1}{\sqrt{1-t^2}} - 2t\arcsin t}{(t^2-1)^2}$$

4. **(5 points)** *A cup of just-brewed coffee has temperature of 90° C and is sitting in a 20° C room. After 10 minutes its temperature has dropped to 75° C. Find a formula $f(t)$ for the temperature of the cup of coffee t minutes after brewing.*

By Newton's Law of Cooling, we know that $f(t) = A + Ce^{-kt}$ for A equal to the ambient temperature — 20° in this case — and for some constants C and k. We also know, from the data in the problem, that the initial temperature $f(0)$ is 90° and that $f(10) = 75$. Plugging in $f(0) = 90$ to the known equation:

$$\begin{aligned} f(0) &= A + Ce^{-k\cdot 0} \\ 90 &= 20 + Ce^0 \\ 70 &= C \end{aligned}$$

and then plugging in $f(10) = 75$:

$$\begin{aligned} f(10) &= A + Ce^{-k\cdot 10} \\ 75 &= 20 + 70e^{-10k} \\ 55 &= 70e^{-10k} \\ \frac{11}{14} &= e^{-10k} \\ \ln\frac{11}{14} &= -10k \\ \frac{\ln\frac{11}{14}}{10} &= -k \end{aligned}$$

so that the final formula is $f(t) = 20 + 70e^{0.1\ln\frac{11}{14}t}$.