1. **(7 points)** *Let $f(x) = 2 + 3x^2 - x^3$. Find the following:*

   (a) **(3 points)** *its critical points.*

   We can determine that $f'(x) = 6x - 3x^2$. The critical points of a function $f(x)$ are those points where its derivative is nonexistent or zero; the former condition will never occur, since $f'(x)$ is a polynomial function and will be defined everywhere, so the critical points are exactly those where $6x - 3x^2 = 0$. Factoring the left side, we see that this equation is equivalent to $-3x(x-2) = 0$, which is satisfied when $x = 0$ or $x = 2$, which will be the critical points.

   (b) **(2 points)** *its maximum and minimum on the interval* $[-2, 1]$.

   We consider all critical points in the interval as well as the endpoints of the interval as prospective extrema on a closed interval. Thus, our candidates here are -2, 0, and 1 (2 is not in this list because it is outside the interval). We calculate $f(-2) = 22$, $f(0) = 2$, and $f(1) = 4$, so -2 is the absolute maximum, and 0 the absolute minimum of this function on this interval.

   (c) **(2 points)** *its maximum and minimum on the interval* $[-1, 3]$.

   We consider all critical points in the interval as well as the endpoints of the interval as prospective extrema on a closed interval. Thus, our candidates here are -1, 0, 2, and 3. We calculate $f(-1) = 6$, $f(0) = 2$, $f(2) = 6$, and $f(3) = 2$, so -1 and 2 are both maxima of this function on this interval, and 0 and 3 are both minima.

2. **(5 points)** *We have a conical pile of sand whose height is twice its radius. The pile of sand is currently of radius 3 inches, and sand is falling on it at a rate of 2 cubic inches per minute. How quickly is the radius of the pile increasing?*

   Let r, h, and V represent the sand pile's radius, height, and volume respectively. We are told that $h = 2r$, and it is a known fact about cones that $V = \frac{1}{3}\pi r^2 h$; using the previous fact, we can eliminate h by noting that $V = \frac{1}{3}\pi r^2(2r) = \frac{2}{3}\pi r^3$. We also know, since the volume of the cone is steadily increasing at a given rate, that $\frac{dV}{dt} = 2$. Frim these facts, we wish to determine $\frac{dr}{dt}$, and we will implicitly differentiate to do so:

$$\begin{aligned} V &= \frac{2}{3}\pi r^3 \\ \frac{d}{dt}V &= \frac{d}{dt}\frac{2}{3}\pi r^3 \\ \frac{dV}{dt} &= \frac{dr}{dt}\frac{d}{dr}\frac{2}{3}\pi r^3 \\ \frac{dV}{dt} &= \frac{dr}{dt}2\pi r^2 \\ \frac{\frac{dV}{dt}}{2\pi r^2} &= \frac{dr}{dt} \end{aligned}$$

   Since we know all the quantities on the left sinde of this equation, we can compute that $\frac{dr}{dt} = \frac{2}{2\pi \cdot 3^2} = \frac{1}{9\pi}$.

3. **(4 points)** *Estimate $\sqrt[3]{0.991}$ using a well-chosen linear approximation.*

We consider the function $f(x) = \sqrt[3]{x}$, whose derivative is $\frac{1}{3x^{2/3}}$. For x close to 1 (as 0.991 is), we can use the linear approximation:

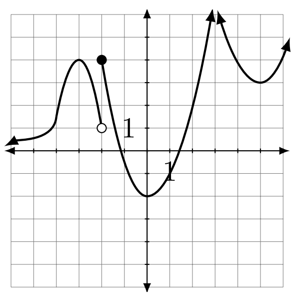
$$f(x) \approx f(1) + (x-1)f'(1)$$

Since $f(1) = 1$ and $f'(1) = \frac{1}{3 \cdot 1^{2/3}} = \frac{1}{3}$, it follows that

$$f(0.991) \approx 1 - 0.009 \cdot \frac{1}{3} = 0.997$$

For purposes of comparison, the actual value of $\sqrt[3]{0.991}$ is around 0.996990954.

4. **(4 points)** *Identify, either by marking them or by giving the x-coordinates, which points on the below graph are local minima and maxima. Also determine which, if any, points on the graph are absolute extrema on $(-\infty, +\infty)$; if none are, then say so.*



By visual identification, we can see that the point at $(-3, 4)$ is higher than anything in its local neighborhood, so it is a local maximum; the same is true of the point at $(-2, 4)$. Note that there is *not* a local minimum at $(-2, 1)$, since $f(-2)$ isn't actually 1. However, we can visually identify a local minimum at $(0, -2)$, and another at $(5, 3)$. There function has no maximum at the x-coordinate of 3, even though this seems like a very high place on the graph, since $f(3)$ doesn't actually exist.

The absolute minimum can be seen to be at $x = 0$, which is in fact lower than any other point depicted on this graph. This graph has no absolute maximum, since we can get aribtrarily large function values by looking at x values either very close to (but not equal to) 3, or x-values which are themselves very large.