Week 1

1.1.13 You put some ice cubes in a glass, fill the glass with cold water, and then let the glass sit on a table. Describe how the temperature of the water changes as time passes. Then sketch a rough graph of the temperature of the water as a function of the elapsed time.

1.1.17. Sketch a rough gaph of the outdoor temperature as a function of time during a typical spring day.

1.1.21. A homeowner mows the lawn every Wednesday afternoon. Sketch a rough graph of the height of the grass as a function of time over the course of a four-week period.

1.1.25. If $f(x) = 3x^2 - x + 2$, find $f(2)$, $f(-2)$, $f(a)$, $f(-a)$, $f(a+1)$, $2f(a)$, $f(2a)$, $f(a^2)$, $[f(a)]^2$, and $f(a+h)$.

1.1.27. Evaluate the difference quotient $\frac{f(3+h)-f(3)}{h}$ for the function $f(x) = 4 + 3x - x^2$. Simplify your answer.

1.1.29. Evaluate the difference quotient $\frac{f(x)-f(a)}{x-a}$ for the function $f(x) = \frac{1}{x}$. Simplify your answer.

1.1.31. Find the domain of $f(x) = \frac{x+4}{x^2-9}$.

1.1.33. Find the domain of $f(t) = \sqrt[3]{2t-1}$.

1.1.35. Find the domain of $h(x) = \frac{1}{\sqrt[4]{x^2-5x}}$.

1.1.37. Find the domain of $F(p) = \sqrt{2-\sqrt{p}}$.

1.1.47. Find the domain and sketch the graph of $f(x) = \begin{cases} x+2 \text{ if } x < 0 \\ 1-x \text{ if } x \geq 0 \end{cases}$.

1.1.49. Find the domain and sketch the graph of $f(x) = \begin{cases} x+2 \text{ if } x \leq -1 \\ x^2 \text{ if } x > -1 \end{cases}$.

1.1.51. Find an expression for the function whose graph is the line segment joining ther points $(1, -3)$ and $(5, 7)$.

1.1.57. A rectangle has perimeter 20. Express the area of the rectangle as a function of the length of one of its sides.

1.1.61. An open rectangular box with volume of 2 cubic meters has a square base. Express the surface area of the box as a function of the length of a side of the base.

1.1.65. In a certain state the maximum speed permitted on freeways is 65 mph and the minimum speed is 40 mph. The fine for violating these limits is $15 for every mile per hour above or below the limits. Express the amount of the fine as a function $F(x)$ of the driving speed.

1.3.31. For $f(x) = x^2 - 1$ and $g(x) = 2x + 1$, find $f \circ g$, $g \circ f$, $f \circ f$, and $g \circ g$, and identify their domains.

1.3.35. For $f(x) = x + \frac{1}{x}$ and $g(x) = \frac{x+1}{x+2}$, find $f \circ g$, $g \circ f$, $f \circ f$, and $g \circ g$, and identify their domains.

1.3.41. Express $F(x) = (2x + x^2)^4$ in the form $f \circ g$.

1.3.43. Express $F(x) = \frac{\sqrt[3]{x}}{1+\sqrt[3]{x}}$ in the form $f \circ g$.

1.3.45. Express $v(t) = \sec(t^2)\tan(t^2)$ in the form $f \circ g$.

1.5.1. Calculate $\frac{4^{-3}}{2^{-8}}$, and simplify $\frac{1}{\sqrt[3]{x^4}}$.

1.5.3. Calculate $8^{4/3}$, and simplify $x(3x^2)^3$.

1.5.19. Find the domain of the function $f(x) = \frac{1-e^{x^2}}{1-e^{1-x^2}}$.

1.5.21. Find an exponential function whose graph passes through $(1, 6)$ and $(3, 24)$.

1.5.23. If $f(x) = 5^x$, show that

$$\frac{f(x+h) - f(x)}{h} = 5^x \left(\frac{5^h - 1}{h}\right)$$

1.5.29. Under ideal conditions a certain bacteria population doubles in size every 3 hours. Suppose there are initially 100 bacteria. Find the size of the population after 15 hours, and give a formula for the population after t hours.

1.6.9. Determine whether the function$f(x) = x^2 - 2x$ has an inverse.

1.6.11. Determine whether the function $g(x) = \frac{1}{x}$ has an inverse.

1.6.13. Determine whether the function $f(t)$ describing the height of a football t seconds after kickoff has an inverse.

1.6.15. Assume f is a one-to-one function. If $f(6) = 17$, then what is $f^{-1}(17)$? If $f^{-1}(3) = 2$, what is $f(2)$?

1.6.17. If $g(x) = 3 + x + e^x$, find $g^{-1}(4)$.

1.6.19. The formula $C = \frac{5}{9}(F - 32)$ expresses Celsius temperature as a function of Fahrenheit temperature. Find a formula for the inverse function and interpret it.

1.6.21. Find a formula for the inverse of $f(x) = 1 + \sqrt{2 + 3x}$.

1.6.23. Find a formula for the inverse of $f(x) = e^{2x-1}$.

1.6.25. Find a formula for the inverse of $y = \ln(x + 3)$.

1.6.35. Find the exact values of $\log_5 125$ and $\log_3 \frac{1}{27}$.

1.6.37. Find the exact values of $\log_2 6 - \log_2 15 + \log_2 20$ and $\log_3 100 - \log_3 18 - \log_3 50$.

1.6.39. Write $\ln 5 + 5 \ln 3$ as a single logarithm.

1.6.41. Write $\frac{1}{3} \ln(x + 2)^3 + \frac{1}{2}[\ln x - \ln(x^2 + 3x + 2)^2]$ as a single logarithm.

1.6.51. Solve $e^{7-4x} = 6$ for x.

1.6.53. Solve $2^{x-5} = 3$ for x.

1.6.57. Find the domain of $f(x) = \ln(e^x - 3)$, and find its inverse.

1.6.61. Find the inverse of the function developed in exercise 1.5.29., and explain its meaning. Then, determine when the bacteria population reaches $50,000$.

1.6.63. Find the exact values of $\sin^{-1} \frac{\sqrt{3}}{2}$ and $\cos^{-1}(-1)$.

1.6.65. Find the exact values of $\arctan 1$ and $\sin^{-1} \frac{1}{\sqrt{2}}$.

1.6.67. Find the exact values of $\tan(\arctan 10)$ and $\sin^{-1}(\sin(\frac{7\pi}{3}))$.

1.6.69. Prove that $\cos(\sin^{-1} x) = \sqrt{1 - x^2}$.

1.6.71. Simplify the expression $\sin(\tan^{-1} x)$.